THE EFFECTS OF UNCERTAINTY IN PUBLIC UTILITY REGULATION

BY

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The purposes of this dissertation are 1) to formulate some uncertainty-related models for both the behavioral problems of input and output distortions and of the peak load problems and 2) to evaluate some of their theoretical implications. Uncertainty and attitudes toward risk are particularly important considerations in decision models of the theory of the firm if they are to be accurate assessments of the real world. Recent studies have shown that traditional economic results do not hold when stochastic components are incorporated into the model. In fact, except for a few generalizations, the implications of uncertainty will depend on just how it is introduced into the model. That is, under a certainty model the choice of a decision variable, either price or quantity, is unimportant. But this decision variable is extremely important to the outcome of the firm under uncertainty because of the asymmetry between the *ex ante* and *ex post* controls.

Specifically, it can be said from quantity-setting and/or price-setting models whose objective functions are

based on the maximization of utility in the von Neumann-Morgenstern sense, that the unconstrained risk-averse firm tends to produce a smaller output than the risk-neutral firm and the risk-taking firm chooses to produce more than the risk-neutral firm. Under the price-setting assumption, we find the unregulated risk-averse firm sets a lower price than its risk-neutral counterpart whereas the risk-taking firm here will set a higher price than the risk-neutral firm. When a regulatory constraint is incorporated into the quantity- and price-setting analysis the theoretical results seem to support the argument that both uncertainty and the attitudes towards risk act together to modify or accentuate the direction of these price and quantity decisions. Yet the Averch-Johnson hypothesis is not primarily an output hypothesis. Their emphasis was on the divergence of the capital-labor ratio at optimality when subject to a rate of return on capital constraint. The impact of uncertain demand on the input ratio of regulated firm will depend on whether these factors of production are substitutes or complementary inputs. The results in this area are unclear. Finally the assumption of uncertain demand is dropped for a random input supply function and this seems to be consistent with the existing theoretical results.

Comparisons between the certainty and uncertainty-related models are made in both an unregulated and regulated context for solutions to the peak load pricing policies. The price, output and capacity solutions of the firm with nonlinear risk

preferences are examined and the possibility of pricing reversals are found to be present. Thus, findings on price, quantity, capacity and input proportions differ when attitudes toward risk are nonlinear and are combined with uncertainty-related assumptions.

CHAPTER 1

INTRODUCTION

In the past decade, the behavior of regulated industries has received a substantial amount of attention from economists. The Averch-Johnson model has been the primary structure used in analyzing firm behavior. Essentially, the Averch-Johnson proposition concerns itself with regulation-induced distortions in input proportions and output decisions created by the use of a rate of return on capital constraint. Theoretical papers, such as Zajac [30] or Baumol and Klevorick [6], stem from this A-J hypothesis and present detailed discussions of input proportions and optimal output decisions of the firm.

Another area of concern for regulated industries has been the peak load pricing problem. The introduction of the modern theoretical analysis of peak load pricing was made by Boiteux [8] and Steiner [24] when both simultaneously, yet independently, demonstrated the nature of the optimal solution to peak load pricing by using marginal cost pricing principles. Williamson [28] advanced their research by generalizing the cost functions and providing more flexibility in the mathematical and geometric techniques required to derive the appropriate solutions. The analysis of peak load pricing to this point has been based on a welfare objective and by incorporating the assumption of constant long-run and short-run costs, the optimal pricing solutions would require higher prices for peak than for offpeak users. Until recently, virtually all of the literature dealing with peak load problems seeks to determine conditions for maximizing social welfare. Although many industries experiencing peak load problems are also regulated monopolies, this aspect has yet to be included in any behavioral model. Bailey [2] derives some peak load pricing principles for the regulated monopoly based on the regulatory constraint and concludes that prices to peak and offpeak users depend on the type of regulatory constraint placed on the firm. Bailey and White [4] propose that the regulated firm may be induced to set higher prices for offpeak users than for the peak demand periods. This is possible if offpeak demand is substantially more inelastic than peak demand; if marginal operating costs are high relative to marginal capacity costs; and/or the enforcement of the particular regulatory constraint is ineffective. Pricing reversals are more probable under a rate of return on capital regulation since the firm best serves its owners by encouraging peak demand use via lowering price, thereby increasing capacity requirements, and raising price to the inelastic offpeak demand to compensate for revenue losses. Thus, Bailey [2] and Bailey and White [4] were the first to incorporate the peak load problem within the behavioral model of the regulated firm. Clearly, the traditional pricing structure for peak and offpeak demands may not hold, particularly under a rate of return on capital regulatory constraint.

Recently economists have been directing more attention to the explicit incorporation of uncertainty into microeconomic models of the firm. Thus, the introduction of uncertainty into the neoclassical theory of the firm may significantly alter the results, especially when the attitude of the firm is taken into consideration. Most earlier inclusions of stochastic components into econometric models were made but the belief persisted that a properly formulated deterministic model would eliminate the need for any stochastic variable. Studies dealing with the uncertainty aspect have been made, notably by Baron [5], Sandmo [21], Blair [7], Horowitz [11], Penner [19], Leland [17], etc. Leland's [17] general framework incorporates a stochastic demand and allows for variations in behavioral attitudes and ex ante controls of price and/or quantity. Most of the earlier studies are, essentially, special cases of Leland's generalized model. These studies have shown that optimal response on the firm facing uncertain demand renders deterministic economic models inadequate explanations of firm behavior. This thesis will extend the basic Leland analysis to the regulated firm and evaluate the optimal responses. The most serious drawback of this uncertainty modification is the inability to obtain unambiguous results when dealing with different attitudes towards risk in the unregulated as well as the regulated case.

The purposes of this dissertation are 1) to formulate some uncertainty-related models for both the behavioral problems of input distortions and of the peak load problems and 2) to evaluate some of their theoretical implications. Uncertainty and attitudes toward risk are particularly important considerations in decision models of the theory of the firm if they are to be accurate assessments of the real world. Chapter 2 surveys several models of the Averch-Johnson hypothesis and presents a brief comparison of the optimal input proportions for the unregulated and regulated firm. Here, demand, cost and production parameters are nonstochastic. Chapter 3 is devoted to developing and extending the deterministic model's optimal input proportions and output responses into the realm of uncertainty. Uncertainty is first expressed as demand uncertainty and analyzed based on the firm's attitude toward risk. Then uncertainty is incorporated into the input prices and the outcome is presented for both the unregulated and regulated case. Chapter 4 describes the traditional peak load pricing literature, and the more recent literature in this area dealing with the imposition of a regulatory constraint. Chapter 5 extends the regulated peak load problem to uncertainty related variables and attitudes toward risk. The optimal responses are then examined and some general observations set forth.

CHAPTER 2

AVERCH-JOHNSON LITERATURE REVIEW

Certain industries in the market structure exhibit a common characteristic which we call natural monopoly. When the minimum optimal scale of production is sufficiently large that to realize production and distribution economies of scale only one firm need enter the market, the industry is defined as a natural monopoly. That is, in these industries, long run marginal cost falls continuously with increases in output which saturates the potential market demand, thereby allowing the monopoly to experience lower marginal costs than would a number of small competing firms. Regulation is imposed to obtain these economies of scale while, at the same time, to prevent the firm from raising prices in order to exploit its monopoly position. The grounds for this exception to the free market structure are fairly well accepted. Although a variety of other reasons exist to justify the regulation of certain industries, the primary purpose for regulating public utilities is to protect buyers from potential monopolistic exploitation. Thus, for public utility industries, monopoly is recognized as the most efficient structure and is allowed and regulated to approximate competitive results.

Generally, regulation is regarded as a restraining influence, a watchdog to assure that the public interest is being protected. Regulation is incapable of assuring a positively good performance in the private enterprise system, but it can exert substantial influence on certain variables within the firm which will affect its profitability and efficiency. The primary variable most commonly accepted and used by the regulatory agency is the firm's rate of return on invested capital. This variable represents the most fundamental determinant of the firm's economic performance and many other variables are indirectly affected by it. Once the rate of return, as defined by regulations, is set, questions arise as to how it affects the firm's behavior and performance. The principal structure used in analyzing this behavior has been the Averch-Johnson model of the regulated firm, which demonstrates that if the fair rate of return is less than the monopolistic rate of return but greater than the marginal cost of capital, the profit maximizing regulated firm will use a capital-labor ratio greater than the one minimizing the cost of output produced and the firm will overinvest in the capital input. Thus, the Averch-Johnson hypothesis provides the explicit comparisons of input and output of the regulated monopolist with that of its unregulated counterpart. Specifically, the Averch-Johnson effect demonstrates that a public utility may make investments, the social benefits of which are less than the social costs, since these investments will expand the rate base. This, in turn, will affect the absolute dollars of profit generated and to the extent net revenues associated with incremental investment falls short of yielding the allowed rate of return, they can be recouped by raising rates in lucrative markets. Thus, public utilities may tend to adopt excessively capital intensive technology and to take on additional business at unremunerative rates.

The standard A-J model assumes demand and production functions fixed with two inputs, capital and labor, producing a single output. A simple graphical interpretation of this A-J hypothesis is demonstrated below in Figure 2-1.[1]

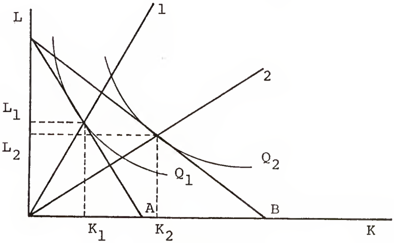


FIGURE 2-1

AVERCH-JOHNSON INPUT DISTORTION

L = quantity of labor input

K = quantity of capital input

A = isocost line of the unregulated firm assuming market costs for capital and labor

B = isocost line of regulated firm assuming the private costs of capital and labor

Q_1 and Q_2 = isoquant curves where output level of $Q_2 > Q_1$

The unconstrained firm travels expansion path 1 as it increases its output since this is where market determined cost is minimized (i.e., where the slopes of the isoquants are equal to the market determined price ratio). With regulation, the cost of capital to the firm is no longer equal to the market cost. Thus, for each additional unit of capital invested by the firm, it will receive both a cost of capital return as well as an additional percentage return per unit based on the difference between the permitted rate of return on investment and the market cost of capital. Clearly, the regulated firm looks upon its private cost of capital as being less expensive and isocost curve B becomes relevant and the regulated firm moves along expansion path 2 where private but not market costs of factor inputs are minimized. This incentive for regulated firms to overcapitalize was demonstrated heuristically in a simple numerical illustration by Scherer [22]. Assume the market cost of capital = 8% and the allowed rate of return = 10%. Also, assume any complications such as depreciation, market imperfections, away. Thus, for every one million dollars of capital invested by the firm, its rate base will increase by one million dollars. Although it will have $80,000.00 more in expenditures, it will be allowed to receive $100,000.00 of additional revenue, the allowed return to capital. Clearly, there is a $20,000.00 bonus for stockholders because of the increased investment in capital made by the regulated firm.

Averch and Johnson [1] concluded that not only does the regulated firm adjust its price ratio because of the rate of return constraint and, thus, substitute capital for labor, but the rate of return constraint will also cause an expansion of the output produced by the firm.[2]

Figure 2-2 illustrates the equilibrium output position of the constrained and unconstrained firm, where MC and AC curves are associated with the unregulated profit maximizing monopolist and where AC' reflects a less efficient expansion path's cost curve when subject to regulation.

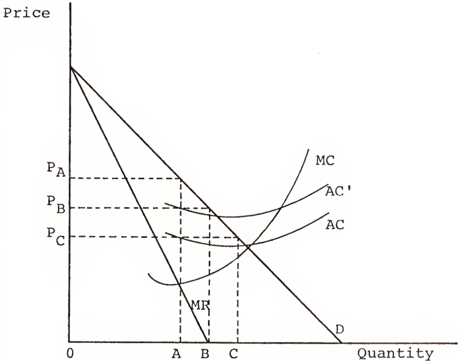


FIGURE 2-2

AVERCH-JOHNSON OUTPUT DISTORTION

The profit maximizing monopolist would produce output OA at a price OP_A, since this is where marginal revenues and marginal costs are equal. For the regulated firm, however, we assume the price to be slightly higher than the average cost to reflect the fact that the permitted rate of return is slightly above the cost of capital.[3] If the regulated firm were to move along the socially efficient expansion path 1, it could produce output OC at a price, OP_C. But Figure 2-1 shows that the firm does not move along the socially efficient expansion path 1 but rather along expansion path 2. The average cost curve will rise (since AC reflected the most efficient path's average cost curve) and the equilibrium output for the regulated firm moving along its chosen expansion path would be output OB at a price OP_B. Thus, the effect of rate of return regulation is to entice the firm into producing more than it would if it were a profit maximizing monopolist but it fails to make or encourage the firm to produce the socially optimal output. At just which point between OA and OC the regulated firm choses to produce will depend largely upon the nature of the production function. Generally, if fixed proportions are necessary in the production process, regulation will not force the firm off the efficient expansion path. The more flexibility the firm has in substituting capital for other factor inputs, the less socially optimal is its chosen output because the greater the possible divergence from the efficient expansion path.

Using the same Averch-Johnson assumptions, Zajac [30] presented a more sophisticated graphical analysis of the A-J effects. Also, assume that for each combination of capital and labor there will be only one value of profit so our three dimensional graph of K, L, π will look like Figure 2-3.[4]

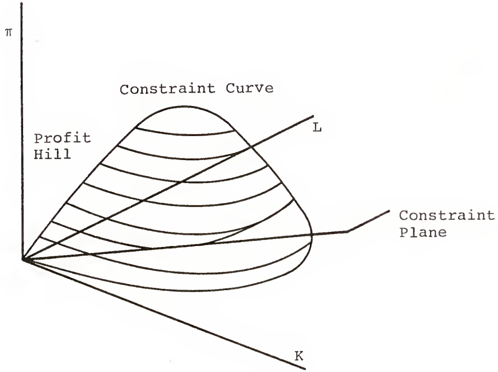


FIGURE 2-3

PROFIT HILL AND CONSTRAINT PLANE

The contour lines on the profit hill are iso-profit (equal profit) lines and the rate of return constraint is a plane hinged on the labor axis. Before proceeding, some additional symbols and algebraic expressions must be identified:

P = price at which each unit of output, Q, is sold

w = wage rate for labor

i = cost of capital (opportunity cost)

s = regulated "fair" rate of return

Output, Q, is given by the production function

$$Q = Q(K,L)$$

The price of each unit of output is given by the inverse demand function

$$P = P(Q)$$

and, of course, profit is defined as

$$\pi = PQ + wL - iK. \tag{2-1}$$

Since Zajac [30] neglects depreciation and capital market imperfections (as did Averch and Johnson [1]), K, the amount of capital used by the firm, will be its rate base. Thus, the rate of return to capital would be total revenue less labor expenses. This return must be less than or equal to the allowable rate of return.

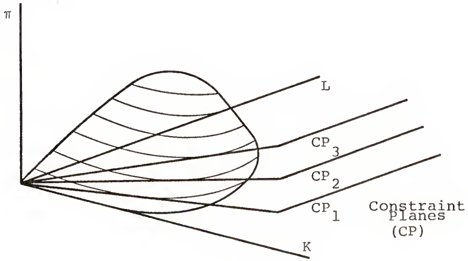
$$\frac{PQ - wL}{K} \leq s \tag{2-2}$$

Combining the profit function with the rate of return constraint we have

$$\pi \leq (s-i)K \tag{2-3}$$

If the "fair" (allowable) rate of return, s, is less than the cost of capital, i, the firm will have negative profits and thus close down. When the "fair" rate of return and the cost of capital are equal, i.e., $s = i$, the constraint is considered perfectly effective and graphically would lie on the K-L plane where there would be no excess returns to capital. As the allowable rate of return, s, increases with respect to the cost of capital, i, the slope of the constraint plane increases. The intersection of the constraint plane with the profit hill will produce an equal rate of return constraint curve when projected onto the K-L plane. The regulated firm's objective is to operate at a point on this constraint curve having highest profit. If the constraint is effective, we can write   = (s-i)K and then the firm will be operating on the appropriate constraint curve projected on the K-L plane in Figure 2-4(b).

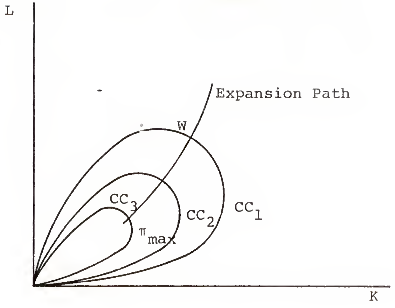
A graphical illustration of how various permitted rates of return appear in a three dimensional K, L,   diagram is provided in Figure 2-4(a). The corresponding constraint curves are shown on a two dimensional K-L plane in Figure 2-4(b).



(a) Constraint Planes Cutting Profit Hill

FIGURE 2-4

PROJECTION OF CONSTRAINT PLANE



(b) Projection of Constraint Curve Costs K-1 Plane

FIGURE 2-4

PROJECTION OF CONSTRAINT PLANE

Thus, any combination of capital and labor in Figure 2-4(a) that generates a profit above the constraint plane is prohibited. Combinations of capital and labor that yield a profit on the constraint plane are maximal and combinations below the constraint plane are inefficient since the firm can improve its profit position by moving towards the constraint plane. The shaded region within the constraint curves of Figure 2-4(b) is the prohibited area where input combinations yield higher rates of return than the maximum permitted by regulation.

For example, assume CP$_3$ (and its corresponding CC$_3$) represent the profit maximizing firm's position at π_{max}. Clearly, then, the greater the difference between the permitted rate of return and the cost of capital, the closer the regulated firm comes to receiving the same rate of return as the profit maximizing monopolist. That is, as regulation becomes increasingly more lax, the size of the prohibited (shaded) area in Figure 2-4(b) becomes increasingly smaller and the rate of return on capital moves closer to the profit maximizing point of the unregulated monopolist. The larger the permitted rate of return relative to the cost of capital, the less effective regulation will be. Conversely, as s approaches i, the constraint plane swings down (e.g., CP_1, the larger the prohibited area becomes, e.g., CC_1).

Using the inverse demand function, P = P(Q), isoquant curves can also be viewed as isorevenue curves and including the profit function of the unregulated firm, it follows that the firm will operate at the point on isoquant where factor costs are minimized. Thus, the locus of efficient points, i.e. the expansion path, of an unconstrained firm can readily be drawn (since isoquants and factor costs are given) and must pass through the profit maximizing point at the peak of the profit hill.

Zajac [30] shows the amount of capital used by the constrained firm to be greater than what the efficient unconstrained firm would require by proving, given an isoquant level and factor costs, that the expansion path cannot intersect this isoquant outside the constraint curve.

Since the definition of the expansion path is the locus of efficient points, it follows that P in Figure 2-5 is a more efficient point of operation for output level, Q, than is K_{max}. Also, recall that in order to meet the capacity conditions, profit decreased as it fell away from the peak of the profit hill. It is easy to see then, that profit also decreases along the locus of efficient points as it moves further away from K_{max}.

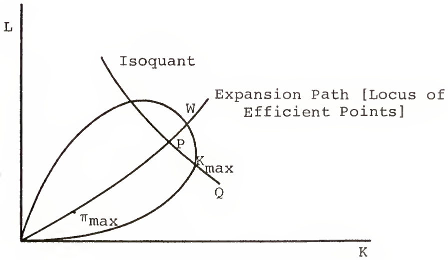
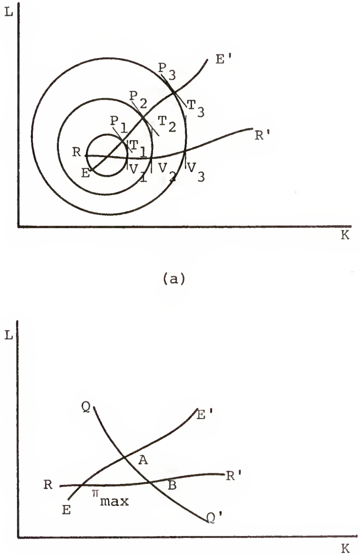


FIGURE 2-5

LOCATION OF EXPANSION PATH

The expansion path must lie above K_{max} and intersect the isoquant (the same isoquant which passes through K_{max}) at a point inside the constraint curve (in the prohibited area of Figure 2-5). Clearly, given the isoquant curve, Q, and the factor prices, costs are minimized at point P. But if it is a regulated firm they will not be allowed to operate here since the firm would be receiving a prohibited rate of return and would be forced to reduce prices of output so as to decrease profits. Ideally, we would like regulation to move the firm to point W and, thus, the regulated firm would substitute capital for labor until it just obtains its allowed rate of return. But since both the rate of return constraint and the factor costs are given, it follows from $\pi = (s-i)K$ that the regulated firm maximizes profits at K_{max}. Hence, Zajac [30] proves that the regulated firm is capital intensive at its chosen output level.

Baumol and Klevorick [6] provide an alternative geometric derivation of the A-J model.[5]



(a)

(b)

FIGURE 2-6

LOCUS OF EFFICIENT POINTS AND EXPANSION PATH OF REGULATED FIRM: ALTERNATIVE GEOMETRIC OUTLOOK

Figure 2-6(a) is an aerial view of Zajac's [30] profit hill with each individually closed curve representing a locus of constant total profit points where the innermost circle depicts the profit maximizing rate of return and each circle thereafter represents decreasing values of allowable rates of return. Both Figures 2-6(a) and 2-6(b) show the locus of efficient points, EE', and the expansion path of the regulated firm, RR'. Figure 2-6(a) shows the combination of inputs chosen by the regulated firm given different values of s, the permitted rate of return.

Baumol and Klevorick [6] note that if the portion of the A-J proposition stating "the capital-labor ratio of the regulated firm will be larger than the one that minimizes cost for the output level that it elects to produce"[6] is true, then the expansion path of the regulated firm, RR', must be below and to the right of the efficiency locus. That is, it must be proven that in Figure 2-6(b) the intersection of the expansion path, RR', with the isoquant, QQ', at point B is below and to the right of point A which is the intersection on the same isoquant but with a different more efficient (by definition) expansion path, EE' (where isoprofit and isocost curves are equal for the unconstrained firm, points T_1, T_2, T_3). Since both the rate of return constraint and the factor costs are given, it follows from $\pi = (s-i)K$, that the expansion path for the regulated firm will be where capital is at a maximum for each isoprofit curve. Thus, the rightmost point of these isoprofit curves in Figure 2-6(a) (where slope is vertical) will be the locus of efficient points to the regulated firm. Given the convexity of the isoquants and, hence, their negative slopes the tangency points, T_1, T_2, T_3 on the negatively sloping portion of these curves must lie above and to the left of its corresponding vertical rightmost points V_1, V_2, V_3.

Mathematical Approach

Baumol and Klevorick [6] develop a more sophisticated mathematical model of the Averch-Johnson hypothesis to show that several false conclusions follow from a naive look at the graphs. The assumptions and symbols developed earlier in the chapter will apply here. Implicit in this model is the notion that the firm can make its own decision on the price-output combination it wants to produce so long as it keeps within the regulatory constraint. It will be assumed further that the rate of return constraint is satisfied as an equality at all times and no regulatory lag exists. This insures an internal solution and obviates the need for the Kuhn-Tucker methodology employed by Averch and Johnson [1]. Clearly, this model of the regulatory process is quite rudimentary. In this A-J model the price setting function of the regulatory commission is lost in the rate of return constraint. In practice, the regulated firm is required to meet the public demand and the price is set so as to obtain the "fair" rate of return, not vice versa. Also, if a regulatory or "time" lag does exist, then the firm may be producing a specified amount of output at an outdated price. That is, the firm is charging a price that once may have yielded the fair rate of return but no longer does due to cost, demand or produnction function changes. Speculation as to the effects of this lag have been intuitively discussed by several authors. Bailey and Coleman [3] developed a model focusing directly on this regulatory lab problem and found that when $s > r$ the incentive to overcapitalize (set forth by A-J) is reduced. With these restrictions in mind the mathematical model can be developed below.

Baumol and Klevorick [6] agree that the regulated profit maximizing firm's input proportions differ from those that minimize the cost of the final output level, Q^*.[7] The input proportions for the unregulated firm subject to a specific production function, $Q^* = Q(K, L)$, can be obtained by cost minimization by solving the Lagrangean

$$\text{(2-4)} \qquad \mathcal{L} = wL + iK + \mu(Q^* - Q(K, L))$$

The first-order conditions are

$$\text{(2-5)} \qquad \frac{\partial \mathcal{L}}{\partial K} = i - \mu \frac{\partial Q}{\partial L} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L} = w - \mu \frac{\partial Q}{\partial L} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = Q^* - Q(K, L) = 0$$

By eliminating the Lagrange multiplier μ, we have

$$\text{(2-6)} \qquad \frac{i}{w} = \frac{\frac{\partial Q}{\partial K}}{\frac{\partial Q}{\partial L}} = \frac{Q_K}{Q_L} = \text{where } Q_K = \frac{\partial Q}{\partial K},\ Q_L = \frac{\partial Q}{\partial L}$$

That is, the ratio of input prices should just equal the ratio of the marginal products of the inputs. Thus, the standard criterion for the minimization of costs for an unconstrained firm chosing its own output level should be $\frac{i}{w} = \frac{Q_K}{Q_L}$. We want to compare this with the input proportions chosen by a regulated firm whose objective is to maximize profits subject to a rate of return constraint. The Lagrange expression for the constrained for is

(2-7) $\mathcal{L} = PQ - iK - wL - \lambda(PQ - sK - wL)$

Differentiating with respect to K and L and rearranging the terms, we have[8]

$$MR_Q Q_K - i - \lambda MR_Q Q_K + \lambda s = 0$$

(2-8) $$(1-\lambda)MR_Q Q_K - i = -\lambda s$$

$$(1-\lambda)MR_Q Q_L - (1-\lambda)w = 0$$

Assuming the interesting case of $s > i$ and $\lambda \neq 1$[9] we have, by dividing and rearranging, the first-order differentials

(2-9) $$\frac{Q_K}{Q_L} = \frac{i}{w} - \frac{\lambda}{1-\lambda}\frac{(s-i)}{w} \text{ where } Q_K = \frac{\partial Q}{\partial L},\ Q_L = \frac{\partial Q}{\partial L}$$

The rightmost term in equation (2-9) must equal zero for the input proportions of the constrained and unconstrained firm to be identical. This would invalidate the A-J proposition of a rate of return constraint causing input distortions which would yield an inefficient use of capital and labor. If the rightmost term does not equal zero, then the A-J proposition is correct and the least cost-input combination is not met by the regulated firm. Thus, in order to prove the A-J proposition dealing with input distortions is valid, one must show the rightmost term in equation (2-9) does not equal zero. Since $s > i$ and $\lambda \neq 1$ are given, the only way this term will be equal to zero is for $\lambda = 0$. Hence, Baumol and Klevorick [6] give us a simple proof that $\lambda \neq 0$.

Compare the profit function

$$\pi = PQ - wL - iK$$

with the Lagrangean function of firm maximizing profit subject to a rate of return constraint:

$$\mathcal{L} = PQ - wL - iK - \lambda(PQ - wL - sK)$$

If $\lambda = 0$ then both the above equations would be the same, thus both the unregulated and regulated firm will have similar profit levels. But it was assumed that a rate of return constraint would reduce the level of profits of the firm which contradicts the possibility of $\lambda = 0$, thus $\lambda \neq 0$.

Although Baumol and Klevorick [6] agree with the A-J proposition that input distortions do exist, they do not believe, as Averch and Johnson, that it necessarily follows that "the capital-labor ratio of the regulated firm will be larger than that of the unconstrained profit maximizing monopolist."[10]

Baumol and Klevorick [6] point out the proposition could be more accurately stated as "the capital-labor ratio of the regulated firm will be larger than the one that minimizes costs for the output level that it elects to produce."[11]

The statement made by Averch and Johnson [1] in their article[12] implies output depends upon substitution and the nature of production function. But their graphical presentation implies that this substitution of capital and labor will always result in an output greater than that of the unconstrained profit maximizing firm (though less than the socially optimal output). Baumol and Klevorick [6] argue that the graphs are misleading and show mathematically that output may or may not be larger than the unconstrained profit maximizing monopolist. With the aid of lemma, $0<\lambda<1$[13] Baumol and Klevorick [6] show that $\frac{i}{w} > \frac{Q_K}{Q_L}$ since the last term in equation (2-9) must be negative when $0<\lambda<1$ for the constrained firm. Since $\frac{i}{w} = \frac{Q_K}{Q_L}$ for the constrained firm the relative use of capital must be greater than the optimal input proportions selected by the unconstrained firm. For the constrained firm, the marginal rate of substitution of capital for labor, MRS_{KL} or $\frac{Q_K}{Q_L}$, is below the ratio of input prices, $\frac{i}{w}$, and since the MRS_{KL} is negative at a fixed output level, then $\frac{i}{w} > \frac{Q_K}{Q_L}$ can only result from a relative increase in the use of capital.

That the regulated firm will produce an output larger than that which maximizes profits is also implied by Averch and Johnson [1] although they make no such specific statement. Both the Baumol and Klevorick [6] and the Takayama [26] analyses disagree with this implication.

They argue because the first-order differential of the constrained firm shows

$$(1-\lambda)MR_Q Q_L - (1-\lambda)w = 0$$

and

$$MR_Q Q_L = w$$

Thus, the marginal revenue product of labor will equal the wage rate. Baumol and Klevorick [6] note that because of this condition, $MR_Q Q_L = w$, the profit maximizing quantity of labor seems to be hired. It would then follow that by combining this quantity of labor with the larger quantity of capital, output would be greater than the profit maximizing output. They go on to note that this does not hold since the quantity of labor hired depends on the quantity of capital and the nature of the production function. That is, this first-order condition does not necessarily require the regulated firm and the unconstrained profit maximizing firm to hire the same quantity of labor. Indeed, the nature of the production function and the substitution effects of capital and labor will determine the quantity of labor to be employed.[14]

The general literature has focused on the input distortions of the A-J effect and only briefly mentions the output effects. With a rate of return constraint there is an inefficient use of inputs (the A-J input distortion) producing cost increases and thus resulting in a contraction of the output level from the socially optimal level. This conclusion is based on the assumption that the firm's objective is to maximize profits. The possibility exists for the firm to adopt objectives other than profit maximizing but they will not be examined in this paper.

NOTES

[1] Wellisz [27] developed a model independent of Averch and Johnson with similar results. The main difference in their articles being Wellisz stressed the output distortions where Averch and Johnson placed their emphasis on the input distortions.

[2] Baumol and Klevorick [6] dispute this, p. 166.

[3] AC includes a "normal" profit but with regulation of the rate of return we have excess profits.

[4] The shape of the profit hill is based on a smooth, continuous demand function and meets the first- and second-order conditions so the surface of the profit hill continuously falls away from the peak. Multipeak situations are plausible but are assumed away because they tend only to complicate the analysis.

[5] The results, however, are in fundamental agreement with those of Zajac [30].

[6] Baumol and Klevorick [6], p. 166.

[7] Baumol and Klevorick [6], p. 165.

[8] $MR_Q = \frac{\partial PQ}{\partial Q}$ = Marginal Revenue of Output.

[9] Baumol and Klevorick [6], p. 166.

[10] Baumol and Klevorick [6], p. 166.

[11] Baumol and Klevorick [6], p. 166.

[12] Averch and Johnson [1], p. 1057.

[13] Proof of this is in Baumol and Klevorick [6], pp. 166-167.

[14] Baumol and Klevorick [6], p. 168 and Takayama [26].

CHAPTER 3

UNCERTAINTY-RELATED INPUT DISTORTIONS

The persistence of the theory of the firm as a deterministic model is slowly being eroded as new evidence has shown that many previously accepted facts of traditional economics are no longer true when random variables are included in the model. This is not to say stochastic components have not been utilized or considered in the past but rather that the deterministic model has, until recently, dominated over any probabilistic analysis. Indeed, the manner of testing empirical data generally included stochastic terms. Yet the earlier inclusion of these stochastic components into econometric models had been made on the basis of probability theory and the belief that eventually a more properly formulated deterministic model would resolve the need for any stochastic terms. This seemingly unprogressive attachment to a deterministic model is perhaps best explained by the belief that the results of a stochastic model and a deterministic model would be essentially the same. Clearly, previous economists viewed any appendages to their deterministic models as ultimately unnecessary. In all fairness it should be pointed out that much of this attitude proliferated because of the use of the representative firm or consumer, i.e., the average firm or consumer. Thus, invoking the law of large numbers would clearly reduce the need for uncertainty variables in an economic model. A brief history of the conflict has been reviewed by McCall [18].

Recent studies dealing with the aspects of decision making with uncertain demand have been made by several economists, notably, Baron [5], Sandmo [21], Smith [23], Horowitz [11], Leland [17] and Penner [19]. Leland's [17] paper is most effective in formulating a general framework incorporating a stochastic demand and allowing for analysis based on behavioral attitudes as well as flexibility in the *ex ante* controls of price and/or quantity. Most of the earlier studies can be derived as special cases from Leland's [17] generalized model.

Thus, the recent works focusing on the firm's optimal response to decisions made when faced with uncertain demand have clearly shown deterministic economic models to be inadequate explanations of firm behavior. The most serious failing of simple modifications of deterministic results is that different attitudes toward risk cannot be handled. In fact, the more general studies have shown that the expected value of marginal revenue may not be equated with marginal costs at the optimum, that fixed costs can have an influence on the short-run output decisions of the firm, and the quantity and price solutions for the firm will differ depending on the *ex ante* controls employed.

Leland [17] has provided the framework necessary to analyze the various optimal responses to random demands in the free, or more appropriately, unregulated market. We want to extend our analysis into the area of the optimal responses made by regulated firms with different attitudes toward risk when faced with uncertain demand. The regulated firm's output and price responses can then do much to extend the well-known Averch-Johnson hypothesis in what we might consider a more validly grounded economic analysis.

The format in this chapter will be similar to Leland's [17] and will include the basic assumptions of a static model in which the firm has a linear demand curve,[1] produces a single output, and knows its cost function with certainty. Because of the uncertainty aspect in the demand, profit is random; thus, profit maximization is an inappropriate objective. As a consequence, it will be necessary to use the von Neumann-Morgenstern utility function and attempt to maximize expected utility. The von Neumann-Morgenstern approach to utility is an extension of the traditional concept of utility as it additionally incorporates uncertain outcomes as objects of the analysis. This type of utility function reflects the decision maker's attitudes toward risk by considering the outcome and the uncertainty influencing it. That is, the decision maker's preference is expressed in an index which is a reflection of the factors influencing its attitudes toward risk. Thus, the standard maximization of profit will be abandoned for the more useful maximization of the expected utility of profit.

Chapter 2 presented a brief comparison of the optimal input proportions for the unregulated and regulated firm set in a deterministic model. Here, the demand, cost, and production parameters were nonstochastic. Now we will move to the unregulated firm facing random demand, and then to the firm facing random demand and also subject to a regulatory constraint. After the equation for computing the input proportion is developed for each circumstance, unregulated and regulated, a detailed analysis outlining the various solutions will follow.

Up to this point, we have considered as stochastic only our linear demand curve but the need for examining the effects of stochastic input prices on optimal responses should not be ignored. Blair [7] recently explored some aspects of this problem in an unregulated firm context. This will be outlined in the chapter and then extended to the regulatory constraint situation for a comparison of the differences in optimal responses.

Unregulated Quantity-Setting Firm Facing Uncertain Demand

In this case, the decision (*ex ante* control) variable is quantity, Q, and price, P, is a random variable with a known (subjective) probability distribution. Assuming that output cannot be produced instantaneously output must be determined before the price is known. The random profit function of this firm will be given by

$$\pi = P(u)Q - C(Q) \tag{3-1}$$

where $P(u)$ = random price of the output,

Q = output, and

$C(Q)$ = total cost of the input.

The firm's objective function is to maximize the expected utility of profit:

$$E[U(\pi)] = E[U\{P(u)Q - C(Q)\}]$$

Differentiating with respect to the control variable gives us the first-order condition for a maximum of expected utility:

$$\text{(3-2)} \qquad \frac{\partial E[U(\pi)]}{\partial Q} = E[U'(\pi)\{P(u) + Q\,\frac{\partial P(u)}{\partial Q} - C'(Q)\}] = 0$$

$$= E[U'(\pi)\{MR - MC\}] = 0$$

We assume the second-order conditions are satified. Using the definition, $E(XY) = E(X)E(Y) + \text{cov}\,(X,Y)$ equation (3-2) can be reduced to

$$E[U'(\pi)]E[MR] + \text{cov}[U'(\pi),MR] - E[U'(\pi)]MC = 0$$

and dividing by $E[U'(\pi)]$ and rearranging gives us

$$\text{(3-3)} \qquad E[MR] = MC - \frac{\text{cov}[U'(\pi),MR]}{E[U'(\pi)]}$$

The risk-neutral firm has a linear utility function meaning $U'(\pi)$ is constant and the covariance between marginal utility and marginal revenue is zero. This firm produces the output where expected marginal revenue equals marginal cost, which is the stochastic analog of the deterministic case. When quantity is the control variable, the sign of the covariance can be determined by evaluating the sign of $\frac{\partial U'(\pi)}{\partial P}$ which is equal to $U''(\pi)\frac{\partial \pi}{\partial P}$ and the sign of $\frac{\partial (MR)}{\partial P}$. The attitude of the firm toward risk determines the sign of $U'(\pi)$. $U'(\pi)$ will be negative for the risk-averse firm and positive for the risk-taking firm since they possess, respectively, concave and convex utility functions. The sign of $\frac{\partial \pi}{\partial P}$ will be positive since as price increase and quantity is set, profits must go up. The sign of $\frac{\partial MR}{\partial P}$ is found below by using $MR = P + Q\frac{\partial P}{\partial Q}$:

$$\frac{\partial (P+Q\frac{\partial P}{\partial Q})}{\partial P} = 1 + Q\frac{\partial^2 P}{\partial Q \partial P} + \frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial P}$$

$$= 2 + Q\frac{\partial^2 P}{\partial Q \partial P}.$$

Because we have assumed linear demand the rightmost term above becomes zero and thus, $\frac{\partial (MR)}{\partial P}$ is positive. Clearly, the risk-averse firm has a negative covariance, which is interpreted in equation (3-3) as an addition to marginal cost, implying quantity will be set where

$$E[MR] > MC.$$

Thus, the risk-averse firm will produce less output than the risk-neutral firm. The reverse holds for the risk-taking, quantity setting firm because its covariance will be positive and thereby reduce marginal costs in equation (3-3) such that

$$E[MR] < MC$$

at optimality. This, of course, means that more output will be produced by the risk-taking firm. These quantity decisions are depicted below in Figure 3-1 where Q_{ra}, Q_{rn}, and Q_{rt} represent desired outputs for the risk-averse, risk-neutral and risk-taking firms, respectively.

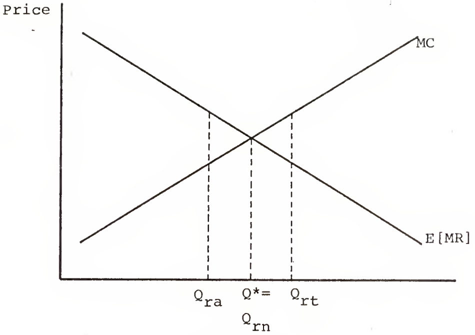


FIGURE 3-1

UNREGULATED QUANTITY-SETTING FIRMS
FACING UNCERTAIN DEMAND

Regulated Quantity-Setting Firm Facing Uncertain Demand

The objective function of the unregulated quantity-setting firm will now be modified to account for an expected rate of return on capital constraint described mathematically as[2]

$$\frac{E[\pi]}{K} = s$$

or as

(3-4) $$\overline{P}Q - C(Q) - sk = 0$$

Price, $\overline{P}$, is such that $\overline{P} = E[P]$ and therefore the constraint is not random. Because random demand precludes an exact constraint, we use an expected constraint. Recognizing that the usual assumptions also apply in this case, the firm's objective is to maximize the expected utility of constrained profits:

$$E[U(\cdot)] = E[U\{P(u)Q - C(Q) - \lambda(PQ - C(Q) - sk)\}] \tag{3-5}$$

The justification for using this particular form is that we are only concerned with feasible solutions which must satisfy the constraint on the average. Substituting the constrained profit definition yields equation (3-5). Differentiating with respect to the decision variable quantity, Q, gives us

$$\frac{\partial E[U(\cdot)]}{\partial Q} = E[U'(\cdot)\{P(u) + Q\frac{\partial P(u)}{\partial Q} - C'(Q) - \lambda(\overline{P} + Q\frac{\partial \overline{P}}{\partial Q} - C'(Q)$$

$$- s\frac{\partial K}{\partial Q})\}] = 0$$

$$= E[U'(\cdot)\{MR - MC - \lambda MR + \lambda MC + \frac{s}{MP_K}\}] = 0$$

$$= E[U'(\cdot)\{(1-\lambda)MR - MC + \frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}\}] = 0$$

$$= (1-\lambda)[E[U'(\cdot)]E[MR] + \text{cov}[U'(\cdot),MF] - E[U'(\cdot)]MC$$

$$+ E[U'(\cdot)](\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K})] = 0$$

$$E[MR] = MC - \frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K} - \frac{\text{cov}[U'(\cdot),MR]}{E[U'(\cdot)]} \tag{3-6}$$

E[U'(·)] on the right-hand side of equation (3-6) is positive because utility is always monotonically increasing meaning U'(·) > 0 and therefore, its expectation is also positive. Since quantity, Q, is given, if price, P, increases, profits will increase on the average. If expected price, $\overline{P}$, in the constraint changes, then the expected constraint must also change as would the analysis. The signs, then, for the covariance will be the same as they were in the unregulated situations. That is, the covariance will be zero for the risk-neutral firm, negative for the risk-averse firm and positive for the risk-taking firm. Therefore, when making comparisons among the various regulated firms based on their attitudes towards risk the same results will hold. The risk-averse firm will produce less output and the risk-taking firm will produce more output than the risk-neutral firm. But the interesting questions arise when evaluating the output decisions for the regulated firms and their various attitudes with their unregulated counterpart.

Baumol and Klevorick [6] prove λ falls within the unit interval making $\frac{\lambda}{1-\lambda}$ positive. Clearly, both s, the fair rate of return on capital, and MP_K, the marginal product of capital are positive. Thus, the regulated risk-neutral firm will tend to produce more output than its regulated counterpart. The value $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ will increase the output level but because of the inverse relationship of marginal product to marginal costs this term also acts to reduce marginal costs and thereby further increase output by some undetermined amount. That is, if $s = r$ then $\frac{s}{MP_K} = MC$ but since $s > r$ we have $\frac{s}{MP_K} > MC$. Although the magnitude of the difference between $\frac{s}{MP_K}$ and MC is not known, several observations can be made. When regulation is effective the fair rate of return, s, will be set close to the cost of capital, r.

In support of this statement, consider the following. If s was substantially higher than r and/or if the parameter, λ, approached 1, then $s = i$ and regulation is rendered virtually ineffective. The value of $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ in equation (3-6) will be larger than the marginal cost, MC, in the same equation and thus, a negative cost situation would exist. Clearly, this would be discarded as an uneconomical and irrelevant consideration. Therefore, it is safe to assume that the value of $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ is something less than the value of the marginal cost in equation (3-6). The risk-taking firm will produce to the right of both the regulated and unregulated risk-neutral firm as well as the unregulated risk-taking firm as it perceives $E[MR] < MC$. Figure 2-3 below depicts the output decisions of these firms.

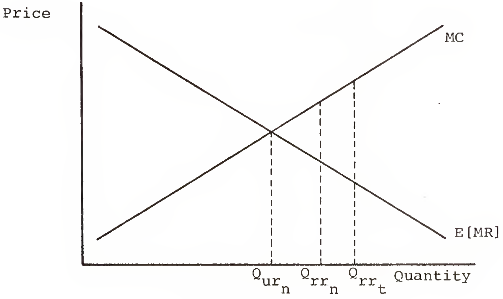


FIGURE 3-2

REGULATED QUANTITY-SETTING FIRM
FACING UNCERTAIN DEMAND

where

Q_{ur_n} = output decision with uncertainty demand conditions for unregulated risk-neutral, and

Q_{rr_n} and Q_{rr_t} = output decision under uncertainty demand conditions for regulated risk-neutral and risk-taking firms, respectively.

Because the rightmost term on the right-hand side of equation (3-6) becomes an addition to the marginal cost whereas $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ tends to reduce the marginal costs, the output decision for the regulated risk-averse firm is indeterminate. When the absolute value of $\frac{\text{cov}[U'(\cdot),MR]}{E[U'(\cdot)]}$ is greater than $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$, the regulated risk-averse firm will select an output which is less than the output produced by the risk-neutral firm. (Again, this output reduction is modified by some undetermined amount because of the inverse relationship of marginal product to marginal cost.) If the reverse holds, then the output decision of the risk-averse firm may be greater than the regulated risk-neutral firm.

Furthermore, the more effective the regulatory constraint, the closer λ is to 0, and thus $\frac{\lambda}{1-\lambda}$ tends to 0. This causes $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ to become increasingly smaller or fall out of equation (3-6) altogether. The more effective the regulatory constraint is, the less divergence there is in the quantities set by the regulated and unregulated firms for all risk preferences. That is, equation (3-6) approaches the value of equation (3-3) as $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ approaches 0.

Thus, the Averch-Johnson hypothesis that the regulated firm will produce an output which is greater than that which maximizes profits (in the unregulated risk-neutral or deterministic model) is less general than they had supposed. The attitude of the firm towards risk will play an important role in the output decision as will the introduction of uncertainty into the demand function. Yet the

Averch-Johnson hypothesis is not primarily an output hypothesis. Their emphasis was on the divergence of the capital-labor ratio at optimality when subject to a rate of return on capital constraint. The following section evaluates the impact of uncertain demand on the input ratio of the regulated firm.

Optimal Selection of Factors of Production Given the Regulated Firm and Uncertainty

The implications of uncertainty when dealing with the selection of factors of production will depend on the firms attitude towards risk. In this context, it is convenient to assume uncertainty enters through price. Thus, the decisions about factor service levels for the regulated firm will be given by the profit function.

$$\pi = P(u)Q - rk - wL \tag{3-1'}$$

where the expected rate of return on capital constraint is described as

$$\bar{P}Q - sk - wL = 0 \tag{3-4'}$$

As in the earlier regulated model, expected price is used in the constraint and therefore, the constraint is not random. Once again, we maximize the expected utility of constrained profits but now we will differentiate with respect to the capital and labor inputs as they are now the decision (control) variables. Thus, the objective function will be

$$E[U(\cdot)] = [P(u)Q - rk - wL - \lambda(\bar{P}Q - sk - wL)$$

By differentiating with respect to capital, we have

$$\frac{\partial E[U(\cdot)]}{\partial K} = E[U'(\cdot)\{P(u)\frac{\partial Q}{\partial K} + Q\frac{\partial P(u)}{\partial Q} \cdot \frac{\partial Q}{\partial K} - r - \lambda(P\frac{\partial Q}{\partial K}$$

$$+ Q\frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial K} - s)\}] = 0$$

$$= E[U(\cdot)\{MRP_K - r - \lambda(MRP_K) - s\}] = 0$$

$$= E[U(\cdot)\{(1-\lambda)\{MRP_K - r - \frac{(r-s)}{(1-\lambda)}\}] = 0$$

Using the definition of the expected value of a product $E[XY] = E[X]E[Y] + \text{cov}[X,Y]$ gives us

$$(1-\lambda)[E[U'(\cdot)]E[MRP_K] + \text{cov}[U'(\cdot),MRP_K] - E[U'(\cdot)r$$

$$- E[U'(\cdot)]\frac{\lambda(r-s)}{1-\lambda} = 0$$

and rearranging the terms we have

$$(3\text{-}7) \qquad E[MRP_K] = r + \frac{\lambda(r-s)}{1-\lambda} - \frac{\text{cov}[U'(\cdot),MRP_K]}{E[U'(\cdot)]}$$

Differentiating $E[U(\cdot)]$ partially with respect to labor, L, we find

$$\frac{\partial E[U(\cdot)]}{\partial L} = E[U'(\cdot)\{P(u)\frac{\partial Q}{\partial L} + Q\frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial L} - w - \lambda(P\frac{\partial Q}{\partial L}$$

$$+ Q\frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial L} - w)\}] = 0$$

$$= E[U'(\cdot)\{MRP_L - w - \lambda MRP_L + w)\}] = 0$$

$$= E[U'(\cdot)\{(1-\lambda)(MRP_L - w)\}] = 0$$

$$= (1-\lambda)\{E[U'(\cdot)]E[MRP_1] + \text{cov}[U'(\cdot),MRP_L]$$

$$- E[U'(\cdot)]w\} = 0$$

$$(3\text{-}8) \qquad E[MRP_L] = w - \frac{\text{cov}[U'(\cdot),MRP_L]}{E[U'(\cdot)]}$$

Since $E[MRP_i] = E[MR] \cdot MP_i$ where $i = K, L$, the ratio of (3-7) and (3-8) is

$$(3\text{-}9) \qquad \frac{MP_K}{MP_L} = \frac{r + \frac{\lambda(r-s)}{1-\lambda} - \frac{\text{cov}[U'(\cdot),MRP_K]}{E[U'(\cdot)]}}{w - \frac{\text{cov}[U'(\cdot),MRP_L]}{E[U'(\cdot)]}}$$

In the regulated deterministic case, we found the ratio of the marginal products of capital and labor relative to their input prices and made observations about their comparability to the standard unregulated criterion based on what we know about marginal rates of substitution, marginal products and revenues, and input prices. Now, we must evaluate the input price ratio for the firm facing a random demand and a rate of return constraint and its relationship to certainty demand curves in the regulated and unregulated context but under similar other circumstances. The possibility of the firm obtaining the "ideal" input proportion, $\frac{r}{w}$, would be virtually nonexistent. For this to occur, the firm would need to be risk-neutral and either the fair rate of return, s, and the cost of capital, r, would need to be identical, or λ must equal zero. Either one of these last two conditions would make the rate of return constraint ineffective and hence, the difference between an unregulated and a regulated firm becomes indistinguishable. Although a precise value of the terms in equation (3-9) cannot be made, several observations are appropriate.

At the optimum, the amount of capital and labor utilized by the regulated firm facing a random demand will vary from the regulated but certainty demand firm's optimal response by the value of $\mathrm{cov}[U'(\cdot),MRP_i]/E[U'(\cdot)]$ and obviously will differ from the input proportions that minimize costs given the output. Clearly, then, when the rightmost term in equations (3-7) and (3-8) as displayed in (3-9) is zero, the risk-neutral, quantity-setting firm will produce at the same point under certainty- or uncertainty-demand situations. The firm will perceive its equation as[3]

$$\frac{MP_K}{MP_L} = \frac{r}{w} + \frac{\lambda(r-s)}{(1-\lambda)w}$$

Nonlinear risk preferences enter equation (3-9) via the covariance term. Thus, to evaluate the input proportions desired by the risk-averse and risk-taking firm the covariances in equation (3-9) must be examined. The marginal revenue and the expected marginal utility of profit curves must be positive regardless of the firm's risk preference in order for them to be economically relevant. The $\text{cov}[U'(\cdot), MRP_i]$ can be evaluated analytically as $\frac{\partial U'(\cdot)}{\partial F_i} = U'(\cdot)\frac{\partial \pi}{\partial F_i}$ where F is defined as the factor of production and $i = K, L$ and $MRP_i = MR \cdot MP_i = MR \cdot \frac{\partial Q}{\partial F_i}$ making $\frac{\partial (MRP_i)}{\partial F_i} = MR \cdot \frac{\partial^2 Q}{\partial F_i \partial F_j}$.

If the firm were risk-averse, $\frac{\partial U'(\cdot)}{\partial F_i} < 0$ since $U'(\cdot) < 0$ and $\frac{\partial \pi}{\partial F_i} > 0$ and $MR \cdot \frac{\partial^2 Q}{\partial F_i \partial F_j} \begin{matrix} > 0 \\ < 0 \end{matrix}$. When capital and labor are complements, $MR \cdot \frac{\partial^2 Q}{\partial F_i \partial F_j} > 0$ and the firm will prefer to use less than the risk-neutral firm under the same circumstances but more than the unregulated risk-averse firm. If the factors of production are substitutes for each other, the regulated risk-averse firm's input proportions is greater than both the regulated and unregulated risk-neutral firm. For the risk-taking firm, the above results hold in reverse assuming the second order conditions are met for the risk-taking firm. This assumption does not necessarily hold and thus the results here are less clear.

Unregulated Firm with Random Input Prices

Now that we have looked at relative input proportions based on various restrictions but with the assumption of nonstochastic input prices, we can introduce random input prices. In these cases, the unregulated profit maximizing firm's profit function can be written as[4]

$$\text{(3-10)} \qquad \pi = P(Q) \cdot Q - \sum_{i=1}^{n} x_i p_i(x_i)$$

where $P = P(Q)$ is the inverse demand function

$Q = F(x_1, x_2, \ldots, x_n)$ is the firm's production function

P and Q = represent price and output, respectively

p_i = represents the price of the i-th input

x_i = represents the quantity of the i-th input

and we deal with only two inputs, capital and labor, where $i = 1, 2$ represents K and L, respectively. Differentiating equation (3-10) with respect to the inputs will give us the first-order conditions for the unregulated firm facing certainty in both their demand function and their cost function

$$\text{(3-11)} \qquad \frac{\partial \pi}{\partial x_i} = P(Q) + \frac{Q \partial P(Q)}{\partial Q} \cdot \frac{\partial Q}{\partial x_i} - p_i(x_i) + x_i \frac{\partial p(x_i)}{\partial x_i} = 0$$

Thus, solving equation (3-11) determines the profit-maximizing combination of inputs.

If we assume that the firm must commit itself to a given output and input prices are stochastic, we may examine the effect of random input prices. Since the focus here is on the effects of random input prices, the demand and production functions will be assumed nonstochastic. The new input prices are given by

$$p_i = p_i(X_i,u_i) = p_i{*}(X_i) + u_i$$

$p_i{*}(x_i)$ is the input supply function with certainty and u_i is the random variable with all the usual assumptions regarding the probability density and expectations.[5] Also, assume $u_i \neq u_j$.

Once again the standard profit maximization must be dropped, as it has no meaning when profit is random, for the more versatile maximization of the expected utility of profit. Thus, the optimal combination of inputs when input prices are random is given by the first-order conditions.

$$\text{(3-12)} \qquad \frac{E\partial U(\pi)}{\partial x_i} = E[U'(\pi)\{P(Q) + \frac{Q\partial P(Q)}{\partial Q}\,\frac{\partial Q}{\partial x_i} - p_i + x_i\frac{p_i(x_i)}{x_i}\}]$$

Second-order conditions are required to be negative definite and we assume that these conditions are satisfied.

Since $MRP_i \equiv (P(Q) + Q\frac{\partial P}{\partial Q})\,\frac{\partial Q}{\partial x_i}$ and $E[MRP_i] \equiv MRP_i$ because demand is nonstochastic and $MFC_i \equiv p_i + x_i\frac{\partial p_i}{\partial x_i}$, the first-order conditions can be written as

$$MRP_i \cdot E[U'(\pi)] - E[U'(\pi) \cdot MFC_i] = 0$$

$$\text{(3-13)} \qquad MRP_i - E[MFC_i] = \frac{\mathrm{cov}[U'(\pi),MFC_i]}{E[U'(\pi)]}$$

The right-hand side of equation (3-13) will equal zero for the risk-neutral firm meaning that the optimal combination of inputs for the certainty and uncertainty model will be identical. If the firm is either risk-averse or risk-taking an examination of the sign of the covariance must be made in order to determine their optimal input combinations. The sign of $\frac{\partial MFC_i}{\partial p_i}$ is positive because as the input price, p_i, increases the marginal factor costs of that input, MFC_i, will increase. The differential of the marginal utility of profit with respect to the input price can be defined as $\frac{\partial U'(\pi)}{\partial p_i} = \frac{\partial \pi}{\partial p_i} \cdot U''(\pi)$ where $\frac{\partial \pi}{\partial p_i}$ is negative since as the price of an input goes up, profit will go down, *ceteris paribus*. Clearly, the sign of $\frac{\partial U'(\pi)}{\partial p_i}$ will depend on the attitude of the firm towards risk.

The risk-averse firm has a concave utility function meaning that the value of the right-hand side of equation (3-13) is positive and thus

$$MRP_i > E[MFC_i]$$

This means that the firm will produce an output such that the marginal revenue product of each input, i, will be greater than the expected marginal factor cost of that input. That is, the unregulated risk-averse firm dealing with random input prices will produce a smaller level of output. This is depicted graphically below as

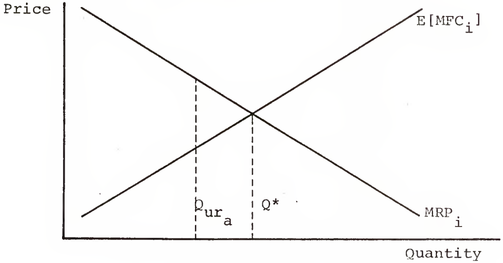


FIGURE 3-3

UNREGULATED RISK-AVERSE FIRM
WITH RANDOM INPUT PRICES

Here Q* is the optimal output for the risk-neutral firm with or without random input prices and Q_{ur_a} is output decision for the unregulated risk-averse firm.

The risk-taking firm's behavior implies a convex utility function with $U'(\pi) > 0$ and combining this with the other terms in will perceive its optimal solution as MFP_i $E[MFC_i]$. Graphically, these relations are depicted in Figure 3-4:

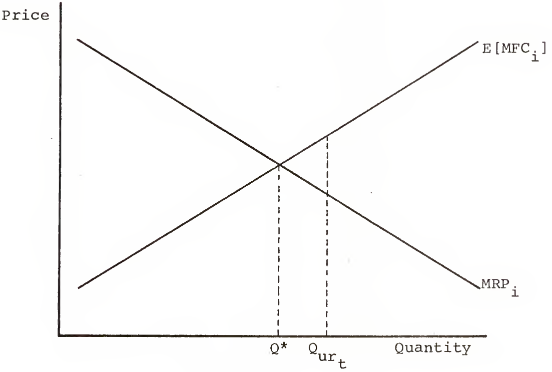


FIGURE 3-4

UNREGULATED RISK-TAKING FIRM
WITH RANDOM INPUT PRICES

Thus, the risk-taking firm will produce more than the optimal output of the risk-neutral firm.

Regulated Firm with Random Input Prices

Extending Blair's [9] analysis a step farther, consider the case of a firm subject to a rate of return on capital constraint. To keep the results of the random input effects clear, continue to assume demand and production functions are nonstochastic. Thus, the firm will once again, maximize the expected utility of constrained profits as given by

$$E[U(\cdot)] = E[U\{PQ - wL - rk - \lambda(PQ - wL - sk)\}]$$

The optimal combination of inputs when input prices are random and the firm is subject to a rate of return constraint are obtained from the first-order conditions for labor and capital:

$$\frac{\partial E[U(\cdot)]}{\partial L} = E[U'(\cdot)\ P\frac{\partial Q}{\partial L} + Q\frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial L} - w - \lambda(P\frac{\partial Q}{\partial L} + Q\frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial L} - w)\}] = 0$$

$$= E[U'(\cdot)\{(1-\lambda)(MRP_L - w)\}] = 0$$

$$= (1-\lambda)[E[U'(\cdot)] \cdot MRP_L - E[U'(\cdot)] \cdot E[w] - \operatorname{cov}[U'(\cdot),w]] = 0$$

$$\text{(3-14)} \qquad MRP_L = E[w] + \frac{\operatorname{cov}[U'(\cdot),w]}{E[U'(\cdot)]}$$

and

$$\frac{\partial E[U(\cdot)]}{\partial K} = E[U'(\cdot)\{P\frac{\partial Q}{\partial K} + Q\frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial K} - r - \lambda(P\frac{\partial Q}{\partial K} + Q\frac{\partial P}{\partial Q} \cdot \frac{\partial Q}{\partial K} - s)\}] = 0$$

$$= E[U'(\cdot)\{(1-\lambda)(MRP_K - r - \frac{\lambda(r-s)}{(1-\lambda)})\}] = 0$$

$$\text{(3-15)} \qquad MRP_K = E[r] + \frac{\lambda E[r-s]}{1-\lambda} + \frac{\operatorname{cov}[U'(\cdot),r]}{E[U'(\cdot)]}$$

The optimal input combination for the regulated firm facing a random input supply is compared to the same situation without the regulatory constraint,[6] the standard deterministic situation, as well as the earlier care of the regulated firm with certainty in the input supply function.[7]

TABLE 3-1

THE UNREGULATED FIRM WITH A CERTAINTY INPUT SUPPLY FUNCTION: [NEOCLASSICAL SOLUTION]

$$MRP_i = MFC_i$$

THE UNREGULATED FIRM WITH AN UNCERTAINTY INPUT SUPPLY FUNCTION:

$$MRP_i - E[MFC_i] = \frac{\mathrm{cov}[U'(\pi), MFC_i]}{E[U'(\pi)]} \tag{3-13}$$

THE REGULATED FIRM WITH A CERTAINTY INPUT SUPPLY FUNCTION:

$$MRP_L = w$$

$$MRP_K = r + \frac{\lambda(r-s)}{(1-\lambda)}$$

THE REGULATED FIRM WITH AN UNCERTAINTY INPUT SUPPLY FUNCTION:

$$MRP_L = E[w] + \frac{\mathrm{cov}[U'(\cdot), w]}{E[U'(\cdot)]} \tag{3-14}$$

$$MRP_K = E[r] + \frac{\lambda E[r-s]}{1-\lambda} + \frac{\mathrm{cov}[U'(\cdot), r]}{E[U'(\cdot)]} \tag{3-15}$$

Special equations for capital are required in the last two situations as the regulatory rate of return constraint alters the input supply function of capital so as to diverge from the input supply function of labor and thus, adds another dimension to the decision making process of the firm.

Regulated Risk-Neutral Firm with Random Input Prices

The covariance in equations (3-14) and (3-15) are nearly identical to the covariance found in Blair [7] and described in an earlier section of this dissertation. Thus, the sign of the covariance for the risk-averse firm will be positive, for the risk-taking firm negative, and, of course, the risk-neutral firm's covariance is zero. The denominator in the rightmost term of equations (3-14) and (3-15) is also the same as the denominator found in the quantity-setting firms subject to regulation. Justification for $E[U'(\pi)] > 0$ was presented earlier in the chapter.

As the covariance term is zero, the regulated risk-neutral firm with random input prices will view its optimal choice of inputs as solving

$$MRP_L = E[w]$$

and

$$MRP_K = E[r] + \frac{\lambda}{(1-\lambda)} E[r-s]$$

Although the first order conditions for labor look the same it would be incorrect to conclude that the firm will employ the optimal quantity of labor because $MRP_L = E[w]$ is consistent with the efficiency requirement of $\frac{MRP_K}{MRP_L} = \frac{r}{w}$ if and only if $MRP_K = r$, which it clearly is not. In fact, $MRP_K = r$ only in the deterministic profit maximizing model. A brief glance at Figure 2-1 readily confirms that the firm in the model will employ an inefficiently small quantity of labor at whatever output it chooses.

Consequently, what is the effect of the rate of return constraint on the firm's labor employment decisions? The amount of labor employed may increase or decrease as s approaches the value, r. If labor and capital are complementary then as the amount of capital employed increases, the amount of labor required will also increase. If capital is a substitute for labor (as a revenue producer, not production function) then capital and labor move in opposite directions.[8]

The amount of capital employed by this regulated, risk-neutral firm will coincide with the Baumol and Klevorick [6] solution for the regulated firm with non-stochastic input prices and thus

$$MRP_K < E[r]$$

What we have then, for the regulated firm with random input supply prices, is the Baumol and Klevorick [6] solution. Therefore, the profits will find that ineffective regulation will lead to the smaller employment of capital.[9] If regulation is effective in that s is set close to r, Baumol and Klevorick [6], Scherer [22] and Takayama [26] show that for some types of production functions, the capital-labor ratio will depart further and further from its efficient value and the quantity of capital will increase. Thus, effective regulation does not assure us of a reduced, absolute capital bias nor does it move the firm towards greater efficiency in the input proportions. But it is important to note here that as s is set closer to r, output will increase as long as capital is not a regressive input.[10]

Regulated Risk-Averse Firm with Random Input Prices

Earlier analysis shows the term containing the covariance to be an addition to the expected wage rate in equation

(3-14) for the regulated risk-averse firm, implying

$$MRP_L > E[w]$$

The amount of labor this firm would employ will be less than the amount of labor employed in either the deterministic neoclassical model, the Baumol and Klevorick [6] model, or the Blair [7] model as expressed in Table 3-1. Clearly, the existence of uncertainty for risk-averse firms will have a dampening effect on the quantity of labor employed, irrespective of any other limiting dimensions (e.g., a regulatory constraint). The quantity of capital desired by this firm would be determined in part by

$$MRP_K = E[r] + \frac{\lambda}{1-\lambda}E[r-s] + \frac{\mathrm{cov}[U'(\cdot),r]}{E[U'(\cdot)]}$$

Comparing this case with the similar but unregulated situation as presented by Blair [7], we find the employment of capital will always be greater than in the unregulated case. When comparisons are made with the regulated but certainty case, i.e., the Baumol and Klevorick [6] model, the firm in question may desire to employ more or less capital depending on whether the absolute value of $\frac{\lambda}{1-\lambda}E[r-s]$ is greater than or less than $\frac{\mathrm{cov}[U'(\cdot),r]}{E[U'(\cdot)]}$, respectively. As regulation becomes more effective, $\frac{\lambda}{1-\lambda}E[r-s]$ falls to zero and the quantity of capital employed will approach that amount desired by the unregulated firm under similar circumstances. Little else can be said definitively regarding the firm's capital employment decisions.

Regulated Risk-Taking Firm with Random Input Prices

Less ambiguity exists for the input decisions of our risk-taker as its situation is expressed by

$$MRP_L < E[w]$$

and

$$MRP_K < E[r]$$

because the covariance has a negative sign. Thus, the risk-taking firm prefers relatively more capital and labor than any of the other cases examined.

Both the regulated risk-neutral and the regulated risk-taking case seem to assure Averch-Johnson overcapitalization. When either of these firms is facing random input supply functions, the response to capital becomes further aggravated. That is, in the case of the risk-neutral and risk-taking firms subject to random input prices, they will tend to employ more capital relative to labor than would that same firm if it was not subject to regulation.

The case of the regulated risk-averse firm facing uncertainty in the input supply function is somewhat different. This firm will employ relatively less labor than the unregulated firm with and/or without random input supply prices. The relative amount of capital used by the regulated risk-averse firm with uncertainty is ambiguous since whether the quantity employed is relatively more than or less than the unregulated firm with uncertainty depends upon the effectiveness of the regulatory agency's rate of return constraint.

Unregulated Price-Setting Firm Facing Uncertain Demand

What happens when the *ex ante* control variable is price, P, and quantity, Q, is distributed around an uncertainty variable, u? We will see the results here differ markedly from our previous solutions when prices were the stochastic components. Again, we assume a linear demand function and set up the profit function as

$$(3\text{-}16) \qquad \pi = P \cdot Q(u) - C \cdot Q(u)$$

and the Lagrangean expression will maximize the expected utility of profits such that

$$E[U(\pi)] = [U\{PQ(u) - CQ(u)\}].$$

In this case, where quantity is random, we assume constant marginal costs. Differentiating with respect to the control variable gives us the first order conditions set forth in equation (3-17) below. Second order conditions are assumed to be satisfied. These assumptions will allow for more definitive results.

$$\frac{\partial E[U(\pi)]}{\partial P} = E[U'(\pi)\{Q(u) + P\frac{\partial Q(u)}{\partial P} - C'(Q)\frac{\partial Q(u)}{\partial P}\}] = 0$$

$$= E[U'(\pi)\{\frac{\partial Q(u)}{\partial P}[P + Q\frac{\partial P}{\partial Q(u)} - C'(Q)]\}] = 0$$

Since demand was assumed linear $\frac{\partial Q(u)}{\partial P}$ is constant and will fall out of the above equation and we may write

$$E[U'(\pi)\{MR - MC\}] = 0$$

or

$$(3\text{-}17) \qquad E[MR] - MC = -\frac{\text{cov}[U'(\pi), MR]}{E[U'(\pi)]}$$

As with the earlier unregulated cases, the introduction of uncertainty does not affect the price-setting, risk-neutral firms pricing decision and it will set price where

$$E[MR] = MC$$

To evaluate the nonlinear risk-taking firm experiencing constant marginal costs it will be necessary to sign the righthandside of equation (3-17). Recall earlier that $E[U'(\pi)] > 0$ has been assumed. The sign of $\mathrm{cov}[U'(\pi),MR]$ can be determined by evaluating the signs of $\partial U'(\pi)/\partial Q$ and $\partial MR/\partial Q$, since quantity is the random variable in question.

The sign of $\partial MR/\partial Q$ is negative since $MR = P + Q\frac{\partial P}{\partial Q(u)}$ and marginal revenue can be expressed as the change in this with respect to output where

$$\frac{\partial(P + Q\frac{\partial P}{\partial Q(u)})}{\partial Q(u)} = \frac{\partial P}{\partial Q(u)} + Q\frac{\partial^2 P}{\partial Q(u)^2} + \frac{\partial P}{\partial Q(u)}$$

$$= 2\ \frac{\partial P}{\partial Q(u)} + Q\frac{\partial^2 P}{\partial Q(u)^2}$$

The second derivative of the linear demand curve is zero making the above equation reduce to

$$\frac{\partial MR}{\partial Q} = 2\ \frac{\partial P}{\partial Q}$$

and since the slope of the linear demand curve is negative, we will have $\frac{\partial MR}{\partial Q}$, negative. It now becomes necessary to determine the sign of $\partial\pi/\partial Q$ where $\frac{\partial U'(\pi)}{\partial Q} = U'(\pi)\frac{\partial\pi}{\partial Q}$. Because we have assumed a constant marginal cost curve and have set prices at a given level, then an increase in output would mean a corresponding increase in profits, i.e., $\frac{\partial\pi}{\partial Q} > 0$. Thus, the attitude of the firm toward risk will determine the sign of $\frac{\partial U'(\pi)}{\partial Q}$. If the firm is risk-averse it will possess a concave utility function meaning $U''(\pi)$ will be negative. This, in conjunction with a positive value for $\frac{\partial\pi}{\partial Q}$ and a negative value for $\frac{\partial MR}{\partial Q}$ will make the sign of $\mathrm{cov}[U'(\pi),MR]$ positive. Again, earlier discussions have already noted $E[U'(\pi)]$ to be greater than zero. Hence, for the risk-averse firm the expected value of the marginal revenue will be less than certainty marginal cost.

$$E[MR] - MC = \frac{-\mathrm{cov}[U'(\pi), MR]}{E[U'(\pi)]}$$

and following this we have

$$E[MR] < MC$$

Thus the price set by the risk-averse firm experiencing constant marginal costs will be less than the risk-neutral firm's pricing decision under similar circumstances. (Since the introduction of uncertainty did not affect the pricing decision of the risk-neutral firm we can say the price set by the risk-averse firm is also less than the price set under certainty, ceteris paribus.)

The risk-taking firm has a convex utility function meaning $U''(\pi) > 0$ and will set price where

$$E[MR] > MC$$

The price set by the risk taking firm will be higher than the price charged by the firm with a risk-neutral behavior pattern.

Relaxation of the Constant Marginal Cost Curve Assumption

Earlier analysis was based on the restrictive assumption that marginal costs were constant. If this assumption is relaxed so the firm can have either increasing or decreasing marginal costs, then the optimal responses of the various firms differ dramatically from their initial position or are impossible to evaluate without more information.

Using Jensen's inequality, Leland [17] proves that the risk-neutral firm with a marginal cost curve rising at a non-decreasing rate will mean an optimal price higher under uncertainty than under certainty. The opposite holds true for the risk-neutral firm with marginal costs decreasing at a non-increasing rate.[11]

The analysis of the risk-averse or risk-taking firm with nonconstant marginal costs becomes even more complicated because the sign of $\frac{\partial \pi}{\partial Q}$ in the covariance term is indeterminate. Although total revenues go up when output is increased, now so does the marginal cost curve.[12] The exact valuation of these components cannot be made in general without more specific knowledge about the demand elasticity. For example, profits will fall if the increase in marginal costs is greater than the increase in total revenues or vice versa. Therefore, no sign may safely be assigned to $\frac{\partial \pi}{\partial Q}$ making any further analysis along these lines unproductive.

Regulated Price-Setting Firm Facing Uncertain Demand

The objective function of the unregulated price-setting firm described in equation (3-17) will now be subject to the rate of return on capital constraint given by

$$P\bar{Q} - C(\bar{Q}) - sk = 0 \qquad (3\text{-}18)$$

Quantity, $\bar{Q}$, is random where $\bar{Q} = E[Q]$. Combining the objective function and the constraint to maximize the expected utility of profits will lead to the following objective function

$$E[U(\cdot)] = E[U\{PQ(u) - C(Q(u)) - \lambda(P\bar{Q} - C(\bar{Q}) - sk)\}].$$

Assuming constant marginal costs and differentiating the above expression with respect to the control variable, P, we find

$$\frac{\partial E[U'(\cdot)]}{\partial P} = E[U'(\cdot)\{Q(u) + P\frac{\partial Q(u)}{\partial P} - C'(Q)\frac{\partial Q(u)}{\partial P} - \lambda(Q + P\frac{\partial Q}{\partial P} - C'(Q)\frac{\partial Q}{\partial P} - s\frac{\partial K}{\partial P})\}] = 0$$

$$= E[U'(\cdot)\{\frac{\partial Q(u)}{\partial P}[P + Q(u)\frac{\partial P}{\partial Q(u)} - C'(Q) - \lambda(P + Q\frac{\partial P}{\partial Q} - C'(Q) - s\frac{\partial K}{\partial Q})\}] = 0$$

$$= E[U'(\cdot)\{\frac{\partial Q}{\partial P}(1-\lambda)[MR - MC + \frac{\lambda}{1-\lambda}\,\frac{s}{MP_K}]\}] = 0$$

$$\text{(3-19)} \qquad E[MR] = MC - \frac{\lambda}{1-\lambda}\cdot\frac{s}{MP_K} - \frac{\text{cov}[U'(\cdot),MR]}{E[U'(\cdot)]}$$

Consider the pricing decision for the risk-neutral firm. Previous discussion has already shown the covariance to fall out and the value of $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ to be something less than MC but also to shift the marginal cost curve. Thus, the price set by the risk-neutral firm experiencing constant marginal costs and a random demand curve will be something less for the regulated firm than for the unregulated firm's pricing position.

Still assuming constant marginal costs, consider the cases of the risk-averse and risk-taking firms. As we noted earlier, the risk-averse firm will have a positive sign for its covariance, thus acting to reduce marginal costs and making the price set by it somewhat lower than the pricing decision of the regulated risk-neutral firm. This, in turn, is something less than it would be for the risk-neutral firm without the regulatory constraint by the value assigned to $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$. A comparison of the pricing decisions of the firm subject to regulatory constraint and its unregulated counterpart can be found by examining the relationship between equations (3-17) and (3-19).

Unregulated:

$$(3\text{-}17) \qquad E[MR]_P = MC - \frac{\text{cov}[U'(\pi), MR]}{E[U'(\pi)]}$$

Regulated:

$$(3\text{-}19) \qquad E[MR]_P = MC - \frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K} - \frac{\text{cov}[U'(\cdot), MR]}{E[U'(\cdot)]}$$

As regulation becomes more effective, equation (3-19) may approach the value of equation (3-17). The price set by the regulated risk-averse firm is lower than the pricing decision of the unregulated risk-averse firm although $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ will act to increase the regulated firm's price by some undeterminable amount. This may even cause a greater price setting decision than its unregulated counterpart if regulation is ineffective. But the more effective the regulatory agency is in enforcing the constraint, the closer the regulated risk-averse price may be to its unregulated pricing decision.

For the risk-taker, the situation is rather ambiguous. As we saw before, the rightmost term will be negative because of the convexity of the firm's utility function and its relationship with $\frac{\partial MR}{\partial Q}$ and $\frac{\partial \pi}{\partial Q}$, and will mean an addition to the right hand side of equation (3-19). If the value of the rightmost term is greater than the value of the second term on the right hand side, $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$, then the price set will be greater than the regulated risk-neutral firm but will be less than the unregulated risk-taker's pricing decision. On the other hand, if the absolute value of the last term in equation (3-19) is not greater than $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ then the value of the right hand side takes on a totally different light and the regulated risk-taking firm will actually set a lower price than the regulated risk-neutral and/or risk-taking firm although not as low as the price set by the unregulated risk-neutral firm. (Here the possibility though slim does exist for the two terms to cancel each other out making the pricing decision for the unregulated risk-neutral and the regulated risk-taking firm identical.) Again the inverse relationship of the marginal product of capital will act to modify these results as it has with the other cases. One point can be made with certainty is that as regulation becomes substantially more effective, the regulated risk-taking firm's price would approach the pricing policy of the regulated risk-taking firm from either above or below, depending on the magnitudes of $\frac{\lambda}{1-\lambda} \cdot \frac{s}{MP_K}$ and $\frac{\text{cov}[U'(\cdot), MR]}{E[U'(\cdot)]}$. Little else can be said about the pricing decisions of the regulated risk-taker. Exactly where their particular price is set relative to other regulated and unregulated firms with the various attitudinal behavior patterns is indeterminate because of the numerous interrelations of the demand curve, cost, and production functions.

Regulated Firms with Increasing or Decreasing Marginal Costs

Earlier the use of Jensen's inequality was employed to show that the risk-neutral firm with a marginal cost curve rising at a nondecreasing rate will mean the setting of a higher price under uncertainty than under certainty. It has been shown that the regulated risk-neutral firm with uncertainty sets its price somewhere below that which an unregulated firm in the same circumstance would do. Thus, extending the analysis above it can be said the regulated risk-neutral firm facing uncertainty and a rising marginal cost curve (as defined above) would set price somewhere below the price set by its unregulated counterpart but somewhere above that price set by the firm facing certainty. The opposite results hold for the regulated risk-neutral firm with marginal costs decreasing at a nonincreasing rate.

Comparative results for the unregulated risk-averse and risk-taking firms with nonconstant marginal costs could not be made in general nor can there be said anything about those firms when a regulatory constraint is in effect. That is, no relations between shapes of cost curves, risk behavior and the effect of uncertainty are immediately apparent.

NOTES

[1] The assumption of a linear demand curve will be used through the entire dissertation.

[2] Again, assume the regulatory constraint is met throughout this chapter and thus eliminating Kuhn-Tucker conditions. Also, for simplification, assume the regulatory lag is nonexistent.

[3] This is identical to the Baumol and Klevorick [6] solution where the second term is shown as $-\frac{\lambda(s-r)}{(1-\lambda)w} = \frac{\lambda(r-s)}{(1-\lambda)w}$, see p. 166.

[4] This notation is taken from Blair [7].

[5] u_i can also enter the input supply function as a multiplicative function instead of additive. Also for specifications of probability densities and expectation, see Blair [7], p. 215.

[6] Blair [7], pp. 220-221.

[7] See Baumol and Klevorick [6], p. 165.

[8] Baumol and Klevorick [6], p. 176.

[9] See Baumol and Klevorick [7], p. 175.

[10] A regressive input is defined as an input such that a rise in its price, ceteris paribus, leads to an increase in the final product, a highly unlikely event.

[11] Leland [17], p. 285.

[12] Leland [17], p. 286.

CHAPTER 4

PEAK LOAD PRICING LITERATURE

When the demand for a service varies appreciably over a period of time, which is short enough to preclude changes in capacity, a problem arises in how best to meet these variations in load and the prices required to accommodate this peaking of demand. This occurrence of peak loads is most commonly found when it is technologically impossible or prohibitively expensive to store the particular product or service. Determining the methodology and technique necessary to formulate the proper pricing structure in order to arrive at some optimum sized plant has been of major concern to those firms and industries subject to demand peaks. The situation is further complicated by the need to incorporate into the framework of the pricing structure not only some optimum sized plant but also the necessary investments and costs associated with it. This problem, in general, is referred to as the peak load pricing problem and traditionally is associated with the case of economies of scale such that because of the declining costs phenomena, prices at or near marginal costs are less than average costs; or when there exists only a partial utilization of total available capacity due to some fraction of the demand cycle requiring a significantly larger amount of demand than any other segment of demand. For now, this essay will deal only with the latter circumstance, i.e., the traditional peak load problem.

Several aspects of the peak load pricing problem may be enumerated. First, technical (brown-outs) and institutional (welfare aspects) reasons make the requirement of sufficient capacity at all times a necessary condition. Because of this requirement for sufficient capacity, the existence of large amounts of idle capacity will result in the offpeak. If the price does not vary between the peak and offpeak users, the demand pattern will consist of intervals of maximum utilization and underutilization. By charging different prices in the various time periods, such that higher prices exist during peak periods to discourage the marginal users and lower prices are used offpeak to encourage the use of the service, a pricing structure could serve to smooth out the demand pattern. Another rationale for a variable price structure is the efficient allocation of resources. The peak demand user incurs both operating expenses and capacity costs upon the system. Offpeak demand customers make no demand on capacity and hence, should not be required to make a capacity cost payment. Rather the offpeak user need only pay a price that covers the operating expenses of providing the output. Thus, using the above price differential would serve to reduce the margin between peak and offpeak demand.

The modern theoretical analysis of the peak load problem was introduced by Boiteux [8] and Steiner [24]. Both authors attempted to show the nature of the optimal solution to peak load pricing by use of the marginal cost pricing policies. Although both set up the same theorem, Boiteux [8] went about proving it using intuitive calculus, whereas Steiner [24] took a geometric approach. Houtthaker's [12] 1951 paper was an empirical study of the supply elasticity of electricity in England. This led to a significent contribution of peak load pricing possibilities by showing that a two rate, time-of-day tariff produced a slight increase in customer costs but an appreciably better use of resources than a tariff based upon a single daily rate. Hirshleifer [10] slightly reformulated Steiner's [24] analysis but differed from Steiner [24] on whether the pricing policies involved price discrimination. In Hirshleifer [10], the allocation of capacity costs to those demands responsible in the shifting peak case (a situation where more than one of the independent demands use the peak load but at different times) was not discriminatory.[1] Williamson [28] generalized the cost functions into which provided more flexibility in both the mathematical and geometric solutions.[2] The generalization of the cost and demand specification makes Williamson's [28] optimality principles for price and capacity readily applicable to any public service which embraces a welfare objective. Thus, the appropriate pricing policy leads to a sufficient amount of physical capacity and its efficient utilization while also covering the full social costs of the resources used.[3]

Most of the analyses of peak load pricing are based on the assumption that the firm exhibits constant long-run and short-run costs and its objective is to maximize welfare. Thus, optimal pricing solutions require higher prices for peak than for offpeak users. Yet many of the industries subject to peak load problems are regulated monopolies and while several behavioral models for those firms exist, particularly the Averch-Johnson [1] proposition, peak load analysis is not incorporated into the analysis in any manner. The same holds in reverse. Bailey [2] derives some peak load pricing principles for the regulated monopoly based on various types of regulatory constraint. The analysis concludes that prices to peak and offpeak demand customers vary with the type of regulatory constraint placed on the firm.

Extending this idea, Bailey and White [4] propose that it may be rational and probable for the regulated monopoly to set higher prices in offpeak vis-a-vis peak demand periods. This would be the case whenever offpeak demand was considerably more inelastic than peak demand. But the pricing reversal is also possible under a rate of return on capital constraint since the firm best serves its owners by encouraging peak demand use by lowering price and thus, increasing capacity requirements. Bailey and White [4] also demonstrate that this regulatory constraint (rate of return on capital) may result in a larger than socially optimal level of capacity. Thus, questions over the necessary amount of physical capacity become more complex and new variables must enter the debate.

Formulation of the Model

At this point it becomes necessary to formulate the assumptions and definitions required in order to establish the basic economic principles for peak load responsibility. These assumptions and definitions will be employed throughout unless otherwise specified. Assume both peak and off-peak users can be served by a uniform type of productive capacity, where short and long run marginal costs, are constant. The firm faces a cyclical demand for its product or service and the cycle can be broken down into n segments. Within each segment the demand, Q_i, associated with a given price, P_i, takes on a constant value which is known with certainty and is independent of any other demand segment. Since each demand segment is not necessarily of equal length, w_i is a fraction introduced to weight each demand segment based on the length of time it prevails over the demand cycle. Thus, each segment of demand, Q_i, and its associated price, P_i, will also be weighted by a fraction, w_i, where the sum of the w_i's over the demand cycle is equal to one.[4] Revenue is then determined by multiplying the price in each period by its respective weighted demand and summing over all segments to find total revenue for the entire cycle:

$$R(Q_i,\ldots,Q_n) = \sum_{i=1}^{n} P_i(Q_i,\ldots,Q_n)Q_i w_i.$$

Costs are separable into two categories. Operating expenses, denoted b, will depend on the total volume of output in each full period (cycle). Capacity costs will depend only on the peak period loads and will be denoted as β, defined as capacity costs per unit per period (cycle). That is, the marginal cost of a unit of capacity is assumed to be constant. For simplicity, we initially assume the existence of a divisible plant so the expansion of capacity will be a continuous process. Plant capacity will be defined in the individual cases that follow.

First, consider the case of the firm peak. This situation will have a fixed and unchanging peak demand where at a specified period of time demand will always press on capacity and any other demand period is never of the intensity required to affect the amount of capacity (offpeak demand). Thus, even when the peak demand bears all the capacity costs, the offpeak demand would not be induced to increased its output.

The objective function for this firm, intent on maximizing profits and experiencing a fixed peak,[5] is given by

(4-1) $\max \pi = P_1Q_1w_1 + P_2Q_2w_2 - bQ_1w_1 - bQ_2w_2 - \beta Q_2$

First order conditions are given by

$$\frac{\partial\pi}{\partial Q_1} = w_1(P_1 + Q_1\frac{\partial P_1}{\partial Q_1}) - bw_1 = 0$$

$$\frac{\partial\pi}{\partial Q_2} = w_2(P_2 + Q_2\frac{\partial P_2}{\partial Q_2} - bw_2 - \beta = 0$$

making

$$P_1 + Q_1\frac{\partial P_1}{\partial Q_1} = b$$

and

$$P_2 + Q_2\frac{\partial P_2}{\partial Q_2} = b + \beta/w_2.$$

Since

$$\frac{\partial P_i}{\partial Q_i} = - \frac{P_i}{Q_i} \cdot \frac{1}{\eta_i},$$

it follows that

(4-2) $$P_1 = b/(1 - 1/\eta_1)$$

and

(4-3) $$P_2 = \frac{b + \beta/w_2}{(1 - 1/\eta_2)}$$

Thus, the pricing solution for the profit maximizing firm consists of third degree price discrimination. Graphically, this can be depicted in Figure 4-1 below:

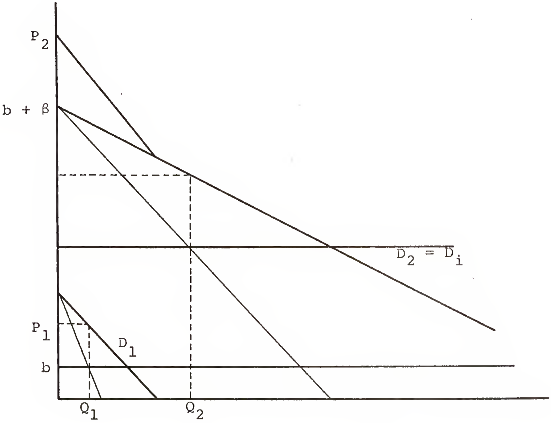


FIGURE 4-1

FIRM PEAK CASE

The offpeak demand, D_1, does not make a demand on plant capacity and should not bear any capacity costs. Any attempt to increase offpeak price above, b, the marginal operating cost, in an effort to get offpeak demand to bear part of the capacity cost will only cause available production capacity at that time to go unused. That is, if you increase price above the marginal operating cost, b, the output demanded in the offpeak will fall and the difference between the available capacity and the amount used in offpeak times will increase. Clearly, this pricing policy would unnecessarily limit the offpeak output since more would be willing to pay the additional costs of serving them. On the other hand, reducing the price for those requiring peak demand service will induce the D_2 users to "stimulate additional purchases at the peak, requiring additions to capacity that would not be made if buyers had to pay the full opportunity costs of additional resources required to supply them."[6]

The shifting peak situation poses a somewhat different interpretation but does not alter the fundamental principle of peak responsibility. In the shifting peak case, instead or only one demand period pressing on capacity, we have two or more potential peak demands. Thus, capacity costs must be allocated between (or among) them. For example, assume that winter and summer customers' demands for electricity are irrevocably peak demands and cause available capacity to be fully utilized during each of these periods. Thus, both customers represent peak, as the same production capacity is available to provide the two separate services (winter and summer); and therefore, both should share in the capacity costs.

To demonstrate this mathematically, assign weights to the revenue and cost functions based on the length of time of the demand in each period, winter and summer. Let

w_1 = winter peak

w_2 = summer peak

where $w_1 + w_2 = 1$

The pricing solution for a strictly shifting peak situation for the firm is given by the solution that optimizes $\pi(Q_1, Q_2)$ using the Williamson capacity definition

$$\pi = P_1Q_1w_1 + P_2Q_2w_2 - bQ_1w_1 - bQ_2w_2 - \beta Q_1 - \beta Q_2. \tag{4-4}$$

Differentiating with respect to Q_1 and Q_2, we find the prices to be given by

$$P_1 = \frac{b + \beta/w_1}{(1 - 1/\eta_1)} \tag{4-5}$$

and

$$P_2 = \frac{b + \beta/w_2}{(1 - 1/\eta_2)} \tag{4-6}$$

If an offpeak demand, say spring customers, is included into the analysis, the objective function of the firm would be given by

$$\pi = P_1Q_1w_1 + P_2Q_2w_2 + P_3Q_3w_3 - bQ_1w_1 - bQ_2w_2 - bQ_3w_3 - \beta Q_1 - \beta Q_2$$

where w_3 = length of spring demand and $w_1 + w_2 + w_3 = 1$.

Then the price will be

$$P_3 = b/(1 - 1/\eta_3)$$

Again, as offpeak customers do not press on capacity no capacity costs should be apportioned to them. The above analysis may be viewed graphically in Figure 4-2 below.

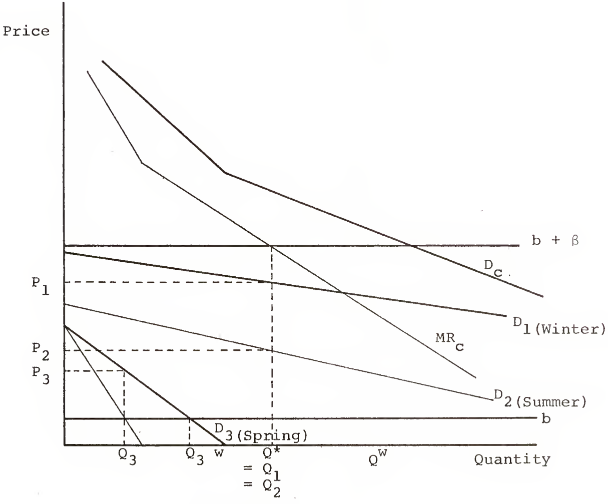


FIGURE 4-2

SHIFTING AND FIRM PEAK CASE

Also, for convenience in the above graph we assumed equal length, independent demand periods. This will generalize the weighting of demand and cost functions but eliminate the necessity of finding an effective demand for capacity curve and instead simply allow the vertical summation of demands. Clearly, the relative elasticities and intensities of each demand are important in determining the prices of the firm intent on maximizing profits. This assumption of independent demands is somewhat tenuous in that it is not unreasonable to assume some cross elasticity of demand in public utilities.[7]

Instead of the profit maximizing criterion, Steiner [24] employs a welfare maximizing criterion such that the optimal amount of capacity is found when the values consumers place on output produced are at least equal to the total cost of producing that unit. In other words, the objective is to maximize the excess of satisfaction over the prices consumers must pay.[8] In Steiner's notation,

$$Z = \overset{i}{\Sigma} F_i(x_i) - b\overset{i}{\Sigma} x_i - \beta\max(x_i)$$

where $F_i(x_i)$ is the valuation placed on the marginal unit of output by customers during the ith period and

$$F_i(x_i) = {}^{x_i}\!\int f_i(x_i)\,dx_i$$

with total costs as

$$TC = b\overset{i}{\Sigma} x_i + \beta x_i$$

x_i represents the number of units of capacity in existence.

Steiner's graphical presentation for both the firm and shifting peak cases are similarly represented by Figures 4-1 and 4-2, respectively.

From a look at the graphs, Steiner [24] assumed the pricing solution of the shifting peak case as discriminatory since the optimal quantities demanded by the winter and summer peak customers in Figure 4-2 would both require the full utilization of capacity yet the prices charged to them would differ.

Hirshleifer [10] contends the correct pricing solution is not discriminatory.[9] Williamson [29], however, more convincingly showed that a look at the graphs and their subsequent analysis can be misleading. In this article, he asserts that the confusion over the issue of price discrimination is due to the intermingling (and hence comparisons) of the pricing solutions of simultaneous demands vis-a-vis the sequential demand situation.

In the case of simultaneous demands, the correct pricing solution for the profit maximizing and welfare maximizing firms would be, respectively:[10]

$$P_1 = \frac{b + \beta}{1 - 1/\eta_1}$$

$$P_2 = \frac{b + \beta}{1 - 1/\eta_2} .$$

and

$$P_1 = b + \beta$$

$$P_2 = b + \beta$$

When sequential demands are utilized in the objective function, the respective pricing solutions for the profit and welfare maximizers are

$$P_1 = \frac{b}{1 - 1/\eta_1} .$$

$$P_2 = \frac{b + \beta/w_2}{1 - 1/\eta_2}$$

and

$$P_1 = b$$

$$P_2 = b + \beta/w_2$$

Thus, the question of price discrimination is best viewed in light of their mathematical formulations. The pricing solution for the profit maximizing firm, whether facing simultaneous or sequential demands, involves the elasticity of demand η_i. Fundamentally, then the price charged is discriminatory. In contrast, pricing solutions stemming from the use of a social welfare objective are based strictly on costs and, thus, are not discriminatory.

Before optimizing the social welfare function, definitions of certain concepts need to be made. Social benefit, SB, is defined as total revenue plus consumer surplus, where consumer surplus is the difference between the consumer's willingness to pay and what the consumer actually must pay. Social cost, SC, the opportunity cost assuming no technological externalities, are divided into total pecuniary costs less intramarginal rent. But for convenience, intramarginal rents are assumed equal to zero.[11]

Following Williamson's [28] notation, the social welfare function is written by

$$\begin{aligned} W &= SB - SC \\ &= S + TR - TC \qquad (4\text{-}7) \end{aligned}$$

where

W = net welfare gain

SB = social benefit

SC = social cost

TR = total revenue

S = consumer's surplus

TC = total costs

Differentiate the original equation (4-7) with respect to output, Q, to derive the first-order conditions:

$$\frac{\partial W}{\partial Q} = \frac{\partial (TR+S)}{\partial Q} - \frac{\partial (TC)}{\partial Q} = 0$$

where

$$TR+S = {}_{O}^{Q}\!\int P(Q')\,dQ'$$

and

$$\frac{\partial (TR+S)}{\partial Q} = \frac{\partial}{\partial Q}\, Q\!\int P(Q')\,dQ'$$
$$= P(Q')$$

Using the cost definitions set forth earlier in the chapter, total costs can be written as

$$TC = (b+\beta)Q$$

making

$$\frac{\partial (TC)}{\partial Q} = b + \beta.$$

Thus, the first-order conditions show

(4-8) $$P(Q') = b + \beta.$$

Second-order conditions must be met where

(4-9) $$\frac{\partial^2 W}{\partial Q^2} = \frac{\partial P(Q')}{\partial Q} - \frac{\partial^2 (TC)}{\partial Q^2} < 0$$

Since $\frac{\partial^2 (TC)}{\partial Q^2} = 0$, condition (4-9) is satisfied if demand is negatively sloped.

Assuming the physical capacity of the plant has already been established, the optimal pricing solution for that capacity must be derived. This is accomplished by maximizing the welfare function subject to $Q \leq \overline{Q}$ where $\overline{Q}$ is the capacity constraint. The Lagrangean is

$$\mathcal{L} = (TR+S) - bQ - \lambda(Q-\overline{Q})$$

Differentiating partially with respect to Q and λ and setting equal to zero

$$P = b + \lambda$$

$$Q \leq \overline{Q}$$

If the constraint is not binding and $Q < \overline{Q}$ is an inequality, then by using Kuhn-Tucker conditions, $\lambda = 0$, and the optimal price would be where output equals the short run marginal costs, i.e. $P = b$. But if the capacity constraint is reached, the value of λ will become positive, making the price greater than SRMC, b. When $b + \lambda > b + \beta$, too much capacity is on hand.

Below is the Williamson [28] review of the case where we have a uniform load and a divisible plant. Marginal operating costs are assumed constant at b per unit per period as long as the output demanded was less than the capacity in existence. He assumes that the output units are efficiently supplied in multiples of size E, an integer at a cost of Γ. Constant returns to scale exist and long run marginal costs are $b + \beta$.

Assume the firm is in equilibrium with demand, D_1, and constant costs, SRMC = LRMC = P. Since LRMC is given by $b + \beta$, price will be equal to $b + \beta$. Optimality requires, $P = b + \beta$. The graphical interpretation is depicted in Figure 4-3 below:

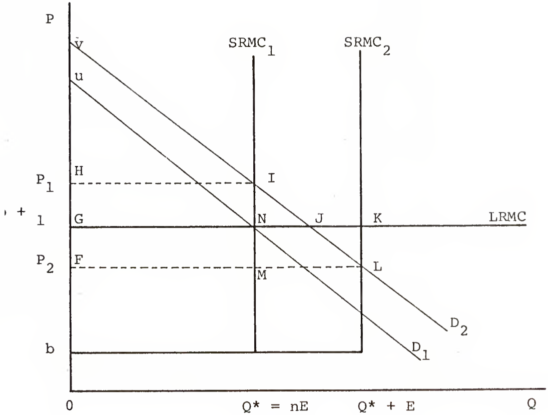


FIGURE 4-3

UNIFORM LOAD AND DIVISIBLE PLANT

Welfare gain is equal to consumer's surplus, UNG, since producer's surplus, TR-TC, is zero.[12]

Suppose the firm experiences a permanent shift in demand from D_1 to D_2. If D_2 is large enough to pass through point K and maintain the long run equilibrium posture, $Q_1^* + E$ units of output will be justified.

The problem arises when the shift in demand is not sufficient to warrant the addition of another efficiency unit, E, to the plant's capacity. The natural question is when can a new unit be installed and what will the optimal price be? To answer this question, assume demand shifts from D_1 to D_2 in Figure 4-3 where the graph is constructed so area IJN equals area JKL. If the firm does not expand capacity beyond the initial Q^* level, it will be necessary to ration output by increasing price to $P_1 = SRMC_1$.

Welfare gain in these circumstances will be the consumer's surplus plus the producer's surplus, VIH + HING = VING. If the additional unit of capacity is added, short run costs will be $SRMC_2$ and the price charged then will be P_2 where $D_2 = SRMC_2$. The consumer's surplus, HING + GJKL + IJN, will just offset the reduction in the producer's surplus HING + GJLF + JKL since IJN is constructed to equal JKL. In this special case, net gain is zero. Thus, the firm is indifferent between maintaining its existing capacity or moving to the new capacity of $Q^* + E$. When IJN becomes greater than JKL the net gain becomes positive and this calls for the addition of the efficiency unit.

Steiner [25] agrees with the Williamson [28] analysis of the indivisible plant problem but points out that private producers are not motivated to provide the increased capacity until demand is sufficiently large so as not to incur any losses. That is, private producers have no incentive to expand capacity until D_2 passes through point K in Figure 4-3.

Williamson [28] acknowledges that the introduction of uncertainty requires the above analysis to be adjusted such that capacity for the risk neutral firm would be added whenever

$$E(IJN) > E[(JKL)$$

The possible application of risk and uncertainty will be explored in the following chapter.

In earlier cases in the peak load pricing problem literature, Boiteux [8], Steiner [24] and Hirshleifer [10] all assume two load periods of equal length. According to Williamson [28], the generalization of this special case may lead to serious errors since analysis of periodic loads requires cost functions and demand functions to be precisely identified. As noted earlier Williamson [28] deals with the entire demand cycle as the specified unit where this cycle could be a day, week, month, season, etc., and costs are defined in terms of the cycle (e.g., a day). This specification then can readily be extended to the multiperiod cases of the peak load pricing problem whereas the analysis made by Boiteux [8], Steiner [24] and Hirshleifer [10] cannot be readily adapted. But Steiner [24] agrues the cost specification problem is negligible since the cycle can be divided into several equal length periods where the summation of consecutive equal length periods can then be designated as peak or offpeak. That is, Williamson's [28] use of β, and b can be defined as $\hat{\beta}$, $\hat{b}$ such that

$$\hat{b}_i = h_i b_i$$

and

$$\hat{\beta} = K$$

where

h_i = number of equal hours in period i, and

K = number of units of output a unit of capacity can provide.

Upon review of the controversy, Williamson [29] realizes the problem does not fall, necessarily, on the cost specifications but rather can be resolved by respecifying the abscissa, the identification of output units. For example, in the Steiner analysis, the two period demands were each one half of the total (fixed) cycle, in the three period, each demand was one third and, thus, the units on the abscissa for each case are not the same, and the output units along the abscissa need to be respecified for each case. This problem then is negligible conceptually and the resulting controversy is of little theoretical importance.

Figure 4-4 below presents the Williamson interpretation of the unequal length demand period problem in more general terms.

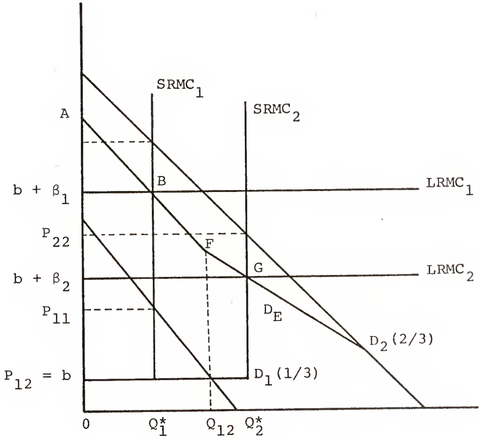


FIGURE 4-4

EFFECTIVE DEMAND FOR CAPACITY

$D_i(w_i)$ expresses demand where i refers to a particular subperiod and w_i to the fraction of the cycle in which that demand prevails. Each demand shows the amount of output per cycle which will be demanded at every price where the demand in question prevails over the cycle. Also assume, for simplicity, independent demands and a divisible capacity.

Using a vertical summation of demands to account for sequential demand, we arrive at an "effective demand for capacity" curve.[13] The resulting effective demand for capacity is shown as D_E.

To prove this construction is correct assume $\beta = \beta_1$ so the long run marginal cost curve, LRMC, intersects the effective demand curve, D_E at point B where optimum output is Q_1^*. Prices will be set where the short run marginal cost curve, $SRMC_1$, meets each periodic demand. Thus offpeak will pay a price of P_{11} for each unit and peak users will pay P_{21}.

Clearly, total revenue equals the sum of $P_iQ_iw_i$ while total costs will be found by summing $(b+\beta_i)Q_1^*$ where w_i = 1/3 and $w_2 = 2/3$. If β_1 (the $LRMC_1$ in Figure 4-4 intersects D_E between points A and F, if capacity is optimally adjusted, and if the offpeak and peak prices, P_{11} and P_{21}, respectively, are equal to their SRMC), then total revenues must equal total costs (net revenue equals zero) since

$$(b + \beta_1) - P_{11} = 2[P_{21} - (b + \beta_1)]$$

That is, the deficit in offpeak period revenue, by not being able to cover pro-rata total costs is just offset by the revenue surplus recouped in the peak period in excess of their pro-rata total costs.

In this graphical analysis where $\beta = \beta_1$, in Figure 4-4 the plant utilizes all of its capacity during both the peak and offpeak demand loads, i.e. a shifting peak case. In mathematical notation, this can be designated by $Q^* = Q_1 = Q_2$ and optimal capacity can be found by making this substitution and differentiating with respect to output, Q^*:

$$W^* = (TR_1 + S_1)\ w_1 + (TR_2 + S_2)\ w_2 - bQ\ (w_1 + w_2) - \beta_1 Q^*$$

$$\frac{\partial W}{\partial P} = P_{11}w_1 + P_{12}w_2 - b(w_1 + w_2) - \beta_1 = 0$$

$$= P_{11}w_1 + P_{12}w_2 - b(w_1 + w_2) + \beta_1$$

and letting $w_1 = 1/3$ and $w_2 = 2/3$ the solution becomes

$$P_{11}\ (1/3) + P_{21}\ (2/3) = b + \beta_1$$

If $\beta = \beta_2$ in Figure 4-5, costs will be lower and the offpeak demand will not press on the existing capacity. Here output will be expanded to Q_2^* units and peak users will be charged P_{22} per unit and demand Q_2^* units of output. Offpeak users pay a price P_{12} (=b) and demand Q_{12} which is less than capacity output, Q_2^*.

The mathematical formulation for the pricing problem in this firm peak case can be written

$$W = (TR_1 + S_1)\ w_1 + (TR_2 + S_2)\ w_2 - bQ_1w_1 - bQ_2w_2 - \beta_2 Q_2$$

Differentiating with respect to Q_1 and Q_2 gives us the first-order conditions for a miximum of social welfare:

$$\frac{\partial W}{\partial Q_1} = P_{12}w_1 - bw_1 = 0$$

$$P_{12} = b$$

and

$$\frac{\partial W}{\partial Q_2} = P_{22}w_2 - bw_2 - \beta_2 = 0$$

$$P_{22} = b + \beta/w_2$$

Revenues and operating costs in the offpeak are equal making it necessary for peak period users to cover their operating expenses as well as the total capacity costs if net revenues are to be zero. Again, construction of the effective demand for capacity, D_E, assures this result.

Also, points B or G in Figure 4-4 can be viewed as the optimal welfare points (with β_1 or β_2, respectively) since any move away from either would reduce net welfare. More specifically, if capacity is expanded, there is a reduction in producer's surplus. If capacity is reduced, the loss of consumer's surplus exceeds the gain in producer's surplus.

Generalizing the mathematics to cover multiperiod loads will give us the following welfare function:

$$W = \sum_{i=1}^{n} (TR_i + S_i)\, w_i - \sum_{i=1}^{n} bQ_i w_i - \beta Q^* \tag{4-10}$$

where

Q^* = optimal plant capacity

Thus $P_i = b$ in periods where plant capacity is not fully utilized and when capacity is fully utilized the pricing solution is

$$\sum_{i=I} (P_i - b)\, w_i = \beta$$

where I is the subset of periods when capacity is fully utilized.

The Integration of Behavioral Models and Peak-Load Analysis

The Bailey [2] and Bailey and White [4] articles were the first to introduce the integration of the peak load problem with the behavioral model of the regulated firm, such as the Averch-Johnson hypothesis. Bailey and White [4] have shown, by the incorporation of these two phenomena into a regulated monopoly model, that the traditional pricing structure for peak and offpeak users may not hold when the firm is subject to some type of regulatory constraint. The natural question to ask then is under what circumstances, i.e., under what type of regulatory constraint, does this pricing reversal take place? Until this point regulatory commissions have been aware of the existence of both behavioral and peak load analyses and theoretically use these as guidelines in the determination of their decision. But if pricing reversals, in fact, exist, and the incorporation of both circumstances under one model are made, then traditional pricing policies for these regulated monopoly firms may no longer be relevant.

To establish a standard of measurement for the various behavioral models and their associated prices and outputs, the Steiner-Williamson analysis will be briefly reiterated here and extended using, essentially, the Bailey-White assumptions and mathematical notation.

Unless otherwise specified, the analyses will be considered as a firm peak situation with two equal length and independent demands. This serves to simplify the analysis and eliminates the need to weight each demand and to apportion capacity costs between (or among) the peak demands. This will in no way alter the analysis. Also, assume the firm experiences constant marginal operating costs, b, as defined in an earlier section of this chapter. β, the earlier notation for capacity costs will be abandoned for the Bailey and White [4] expression, rk as this expression becomes useful when analyzing the A-J constraint possibilities.

The mathematical formulation of the objective function for maximizing welfare (consumer's plus producer's surplus) can be written:

$$\max_{\overline{X},\underline{X}} W = \int_{o}^{\overline{X}} \overline{P}d\overline{X} + \int_{o}^{\underline{X}} \underline{P}d\underline{X} - b(\overline{X} + \underline{X}) - rk\overline{X} \qquad (4\text{-}11)$$

where

$\overline{X}$ and $\underline{X}$ = quantity demanded by peak and offpeak users, respectively,

$\overline{D}$ = peak period demand and is equal to demand for capacity,

$\overline{P}$ and $\underline{P}$ = prices charged to peak and offpeak users, respectively,

b = operating expense per unit of output (constant),

k = acquisition cost per unit of capacity (constant),

r = market cost of capital (constant), and

w = subscript denotes value relating to welfare maximizing objective.

From equation (4-11) it follows that:

$$\begin{aligned} \overline{P}w &= b + rk \\ \underline{P}w &= b \end{aligned} \qquad (4\text{-}12)$$

Thus, peak price covers operating costs plus capacity costs, and offpeak prices (since they make no excess demand on capacity) are required to cover only their operating expenses.

In Figure 4-5 $\bar{P}w > \underline{P}w$ since the welfare maximizing firm would charge peak users a price, $\bar{P}w$, for output, $\bar{X}w$, and offpeak users, $\underline{P}w$ for output, $\underline{X}w$. When costs are constant as in this case, the firm will be in a break even position, i.e., a normal profit position, if they supplied $\bar{X}w$ units of capacity.

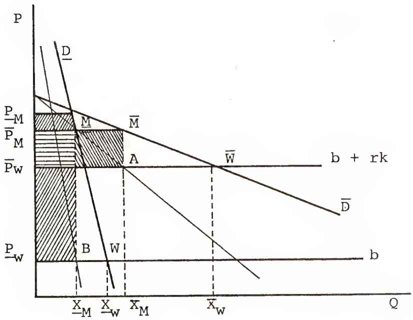


FIGURE 4-5

WELFARE VS MONOPOLY PRICE

The results of a profit maximizing firm vis-a-vis the welfare maximizer are also illustrated in Figure 4-5.

The objective function in this instance will be

$$\max_{\overline{X},\underline{X}} = \overline{P}(\overline{X})\overline{X} + \underline{P}(\underline{X})\underline{X} - b(\overline{X}+\underline{X}) - rk\overline{X} \tag{4-13}$$

where

M = subscript denoting values relating to the profit objective of the monopoly firm.

$\overline{\eta}$ and $\underline{\eta}$ = price elasticities, respectively.

Differentiating with respect to $\overline{X}$ and $\underline{X}$ we obtain:

$$\begin{aligned} \overline{P}_M &= \frac{b + rk}{1 - 1/\overline{\eta}} \\ \underline{P}_M &= \frac{b}{1 - 1/\underline{\eta}} \end{aligned} \tag{4-14}$$

If price elasticities for peak and offpeak users are equal, then the offpeak prices charged will always be lower than the prices charged to peak users. But it is quite possible that the demand in offpeak periods is sufficiently more inelastic than peak periods to reverse the typical pricing policies of the standard welfare maximizing model. Not only will relative demand elasticities be important in possible pricing reversals, but the relative size of marginal capacity costs to marginal operating costs may also contribute to this reversal. That is, offpeak prices will be higher than the peak prices whenever

$$b\left[\frac{1}{1 - 1/\underline{\eta}}\right] > (b+rk)\left[\frac{1}{1 - 1/\overline{\eta}}\right] \tag{4-15}$$

An example, illustrated in Bailey and White [4], is the railway's value of service rates, or in a public utility where demand consists of a larger number of small users along with industrial users who are able to generate their own electricity, if necessary. It is possible for offpeak users with a relatively and sufficiently higher value of service to be charged a price greater than its peak period users whose demand is more elastic. Peak and offpeak consumers will require outputs of $\overline{X}_M$ and $\underline{X}_M$, respectively, at prices $\overline{P}_M$ and $\underline{P}_M$, respectively as shown graphically in Figure 4-5.

Returning to the welfare objective of equation (4-11), if the assumption of constant returns to scale is replaced by an increasing returns to scale possibility (since many industries with peak-load pricing problems also experience scale economies), the results are interesting.

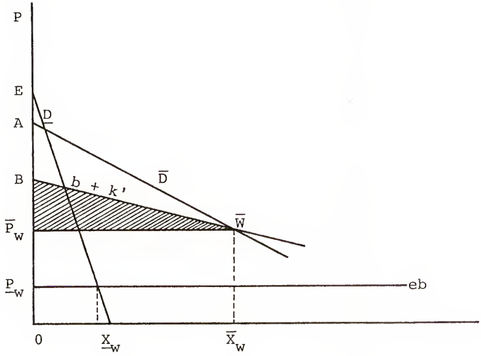


FIGURE 4-6

INCREASING RETURNS TO SCALE

In this case assume capacity cost is given by $K(\bar{X})$, instead of $rk\bar{X}$ in equation (4-11) and the necessary and sufficient conditions, $K'(\bar{X}) = 0$ and $K''(\bar{X}) < 0$ are met. When welfare maximization is the firm's objective, the outputs $\bar{X}w$ and $\underline{X}w$ at prices $\bar{P}w$ and $\underline{P}w$, respectively, are required, such that

$$\underline{P}w = b \tag{4-16}$$

$$\bar{P}w = b + K'(\bar{X})$$

Bailey and White [5] point out that with this pricing policy, the firm will operate at a loss (as shown by the shaded area $B\overline{WP}w$ in Figure 4-6). If the firm is to break even, and if it is to charge only one price for each demand period, then it will be necessary to incorporate a "no loss" constraint into the model

$$\text{Max } W = \int_{0}^{\bar{X}} \bar{P}d\bar{X} + \int_{0}^{\underline{X}} \underline{P}d\underline{X} - b(\bar{X}-\underline{X}) - K(\bar{X}) \tag{4-17}$$

subject to

$$\bar{P}(\bar{X})\bar{X}+\underline{P}(\underline{X})\underline{X}-b(\bar{X}+\underline{X})-K(\bar{X}) \geq 0 \tag{4-18}$$

From the first-order conditions one can arrive at a pricing policy where

$$\bar{P}_I = \frac{(1+\lambda)(b+K')}{1+\lambda(1 - 1/\bar{\eta})} \tag{4-19}$$

$$\underline{P}_I = \frac{1+\lambda(b)}{1+\lambda(1 - 1/\underline{\eta})}$$

Again, it can be seen that price is simply a deviation from marginal cost based on a ratio of the relative demand elasticities.

From the above pricing rules, it is possible for

$$\underline{P}_I > \bar{P}_I$$

Thus, the explanations of relative demand elasticities that are relevant to the previously discussed profit maximizing firm also account for the possible pricing reversals seen by the welfare maximizing firm with increasing returns to scale.

Above is a brief review of the Bailey and White [4] exposition on the peak load pricing problem as it relates to the profit and the welfare maximizing firms. Bailey and White [4] also introduce a regulatory constraint to the profit maximizing firm experiencing a peak load pricing problem.

Using the original Averch and Johnson assumption that the allowable rate of return, s, is greater than the market cost of captial, r, but less than the profit maximizing rate of return, a set of prices can be obtained using the model

$$(4\text{-}20) \qquad \max_{X,\underline{X},Z} \pi = \overline{P}(\overline{X})\overline{X}+\underline{P}(\underline{X})\underline{X}-b(\overline{X}+\underline{X})-rk(\overline{X}+Z)$$

subject to:

$$(4\text{-}21) \qquad P(\overline{X})\overline{X}+P(\underline{X})\underline{X}-b(\overline{X}+\underline{X})-sk(\overline{X}+Z) \leq 0$$

Formulating a Lagrangean function where

λ = multiplier associated with the rate of return constraint;

Z = number of units of capital waste;

G = (subscript) solution prices for regulated model;

and differentiating, Bailey and White [4] find the prices for peak and offpeak consumers in a regulated context to be

$$\overline{P}_G = \frac{b+rk - \frac{\lambda}{1-\lambda}(s-r)K}{1 - 1/\eta}$$

(4-22)

$$\underline{P}_G = \frac{b}{1 - 1/\eta}$$

Since offpeak demand in both the regulated and unregulated context is independent of the capacity cost and the variables r, s, and K associated with capacity cost, it is not surprising to find the offpeak price of the profit maximizing firm subject to a rate of return constraint, precisely the same as the unregulated profit maximizer's offpeak price. Thus, the effect of regulation will be seen only in the price of peak users.[14]

When comparing the price solution for peak users under regulation, $\overline{P}_G$ where $\overline{P}_G = \frac{b + rk}{1 - 1/\eta} - \frac{\frac{\lambda}{1-\lambda}(s-r)K}{1 - 1/\eta}$ with the unregulated welfare maximizing peak users, $\overline{P}_m$ where $\overline{P}_m = \frac{b + rk}{1 - 1/\eta}$, it follows that $\overline{P}_G < \overline{P}_m$ because $- \frac{\frac{\lambda}{1-\lambda}(s-r)K}{1 - 1/\eta} < 0$. That is, Bailey and White [4] view the regulated firm as having an incentive to pass on benefits in the form of price reductions to peak period users whose demand is directly related to the need for increased capacity requirements. Moreover, the possibility of a price reversal in the regulated firm is greater than that outcome in the unregulated monopoly.

The problem now is to find how much capacity is required by the regulated profit maximizing firm in order to capture its total allowed profits. Since we are dealing with a firm peak case we know the offpeak price is readily identified in

Figure 4-7 as $\underline{P}_G$ where marginal revenue is equal to marginal operating costs, b. The surplus profits, $\underline{P}_G\underline{GHP}_W$, associated with this offpeak can be used to subsidize the peak demand, thereby lowering the peak price and increasing the quality (which is equal to capacity). Clearly, peak period capacity is artifically determined in such a manner as to increase the rate bases while at the same time keeping profits down.

For example, if the offpeak period profit is large, the regulated firm would have an incentive to incur a deficit on its peak period operations by reducing the price charged to its peak users below the b+rk requirement. This deficit would offset the offpeak period surplus and simultaneously expand the rate base, and hence expand the total allowable profits. Obviously, price to peak period users would continue to fall until offpeak surpluses just equaled peak deficits.

It is helpful to view this pricing and output solution graphically:

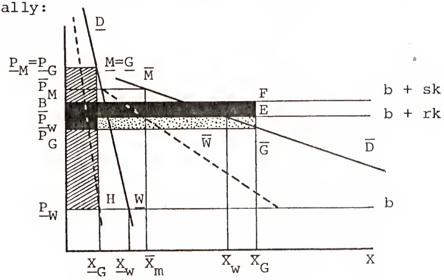


FIGURE 4-7

REGULATED VS MONOPOLY VS WELFARE PRICE

$\overline{M}$ and $\underline{M}$ = (subscripts) denoting values of peak and offpeak periods, respectively for the profit maximizing monopoly;

$\overline{W}$ and $\underline{W}$ = (subscripts) denoting values of peak and offpeak periods, respectively, for the welfare maximizing firm;

$\overline{G}$ and $\underline{G}$ = (subscripts) denoting values of peak and offpeak periods, respectively, for monopoly firm under a rate of return constraints.

Lines b+sk and b+rk are constructed so the difference between them will be (s-r)K.

Again, since $\underline{P}_{\underline{G}}$ is predetermined and results in an offpeak surplus of $\underline{P}_{\underline{G}}GH\underline{P}_{\underline{w}}$, which is greater than the allowed profits $\overline{P}$wBFE, the firm needs to set the peak period price below b+rk so the deficit is sufficient to offset surplus. That is, $\overline{P}_G$ will incur deficit of $\overline{P}wE\overline{GP}_a$ so $\underline{P}_{\underline{G}}GH\underline{P}_{\underline{w}}-\overline{P}_{w}E\overline{GP}_{G} = \overline{P}_{w}BF\overline{E}$.

These reductions in peak period prices are obtained by modifying the monopoly requirement of MR=MC to the regulated monopoly result of MC = adjusted MC where adjusted MC < actual MC. Clearly, regulation induces output to be increased from the monopoly level, but the benefits of this increased capacity accrue only to the peak period users, i.e., those who make the demand on capacity. Thus, the required capacity of a regulated monopoly, $\overline{X}_G$, may exceed the welfare maximizing level, $\overline{X}_m$, whenever the offpeak period surplus is sufficiently large to necessitate peak period pricing deficits.

As s is set closer to r, making b+sk in Figure 4-7 shift downward, the allowed profit level falls. Since this shift has no effect on the offpeak price and surplus associated with it but does reduce the amount of allowable return, the firm will be induced to further reduce the price to peak users and thereby expand its capacity. That is, if allowed profits are reduced while offpeak surplus profits remain constant, then the peak deficits must increase to stay within the prescribed constraint. When the margin narrows because r is increased, making b+rk shift upward in Figure 4-7, the peak period capacity will not change but the welfare maximizing level of capacity will fall. As b+rk shifts upward reducing the allowed profits, while offpeak surplus profits remain constant, the reduction in allowed profits must just offset the increased deficit associated with reduced peak prices bringing capacity requirements to its initial position, $\overline{X}_G$. Since the welfare maximizing firm equates $\overline{P}_W$ = b+rk (from equation 4-12) it will experience a drop in the capacity it demands.

If surplus profits were nonexistent, the regulated firm's peak period would be set where $\overline{P}_G$ = b+sk and price and capacity would fall somewhere between the monopoly and welfare maximizing levels. If s=r, clearly the welfare maximizing and regulated price and capacity would be the same. If s were set so high as to be an ineffective constraint, the regulated firm will act as a profit maximizer.

Possible extensions regarding the various types of constraints under which the monopoly firm can be regulated are reviewed by Bailey [2] in her 1972 article. Briefly, she notes that the method of regulation can affect the distribution of price reductions between peak and offpeak users. For instance, solutions for prices and outputs under rate of return regulation were reviewed above earlier. Under a "fair profit per unit" regulatory procedure, both peak and offpeak period demands will have a lower marginal revenue than will the unregulated monopoly. Thus, price reductions will be shared by both peak and offpeak. In the "fair return on costs" case, peak and offpeak users will receive an across the board percentage decrease in their respective prices.

NOTES

[1] Years later, Steiner [25] revisited the argument and came to agree with Hirshleifer [10].

[2] Williamson [28], pp. 816-824.

[3] This specification problem is dealt with in more detail later in the chapter.

[4] For example, if the demand cycle is defined as twenty-four hours and a particular demand, D_1, at a given price, P_i, prevails for six hours of that day, then w_1 - 1/4.

[5] Fixed peak implies only one demand period pressing on capacity.

[6] Kahn [16], p. 90.

[7] Pressman [20] reaffirms that the assumption of independent demands does not alter the essence of the conclusions.

[8] The Steiner and Williamson definitions are interchangeable with the exception of capacity cost, β. Williamson's capacity cost, β, is defined as the cost of producing a unit of capacity per cycle whereas Steiner defines β as the cost of producing a unit of capacity per subperiod. Therefore, Steiner's capacity costs will be greater than or equal to one depending on the number of subperiods involved in making up the entire demand cycle. Williamson's capacity cost, β, will equal one and the peak demands will be allocated a fraction of this cost. Unless otherwise specified, Williamson's definitions will be used.

[9] By dropping the assumption of constant short run marginal costs for an increasing short run costs, the output demanded in each period such that winter customers will demand X_1 units of output at P_1, and summer customers will demand X_2 at P_2 both of which are something less than available capacity X_0 in Figure 4-2. In this case then different prices would be associated with different outputs.

[10] The mathematical derivation for these pricing solutions can be found in Williamson [29].

[11] Williamson [28], p. 811.

[12] The assumption of a constant marginal utility of money is made to simplify matters.

[13] For a detailed description on how the periodic loads and the effective demand for capacity curve are constructed, see Williamson [28], pp. 818-819.

[14] The possibility exists that the firm may not operate on the production frontier, i.e., $Z = 0$, when it is subjected to regulation but this will not be examined. For more information on this, see Bailey and White [4], p. 81.

CHAPTER 5

UNCERTAINTY-RELATED PEAK LOAD PROBLEMS

Introductory Remarks

Chapter 4 surveys the peak load pricing literature when conditions of certainty were present in both the demand and cost functions of the firm. This traditional peak load problem was further advanced by Bailey and White when they added a regulatory rate of return constraint. The addition of this constraint in the model proved interesting in that it demonstrated the possibility of pricing reversals in the regulated firm.

An extension of the peak load problem is set forth here by abandoning the certainty assumption for an uncertainty variable in our demand function. Comparisons between the unregulated certainty and uncertainty situation will be made for the firm peak case intent on maximizing profits and then for the shifting peak firm under the same conditions.[1] After the possible price, output and capacity solutions have been examined, a rate of return constraint will be added to the model. As anticipated, the possible pricing reversals are present here in varying degrees of probability. We also find that in the firm peak case, offpeak demand plays an important role in determining peak price and capacity.

This is not so in the regulated shifting peak case. Here capacity and price are set where aggregate demand equals the aggregate marginal costs (operating and capacity).

The question of just what effect and how much effect uncertainty has on the price, output and capacity decisions of the firm with nonlinear risk preferences will be taken up later in this chapter.

The following presentation assumes that there are two independent demands of equal length.[2] In the firm peak case, we must assume that a distinct peak and offpeak period exists and that even when subject to uncertainty, its probability distributions are disjoint. This restriction becomes necessary when dealing with uncertainty so the firm peak case does not become a shifting peak problem. These simplifying assumptions are made to facilitate the graphical representations and to focus on the pricing and capacity solutions.

The Model

Below, we have the random profit function for the firm peak case under uncertainty

$$\pi = P_1(u)Q_1 + P_2(v)Q_2 - b(Q_1+Q_2) - rkQ_2 \tag{5-1}$$

Substituting this into $E[U(\pi)]$ we have

$$\max E[U(\pi)] = U[P_1(u)Q_1 + P_2(v)Q_2 - b(Q_1+Q_2) - rkQ_2]$$

Differentiating with respect to Q_1, we have

$$\frac{\partial E[U(\pi)]}{\partial Q_1} = E[U'(\pi)\{P_1(u) + Q_1 \frac{\partial P_1(u)}{\partial Q_1} - b\}] = 0$$

$$= E[U'(\pi)\{MR_1 - b\}] = 0$$

$$= E[U'(\pi)E[MR_1] - E[U'(\pi)]b + \mathrm{cov}[U'(\pi),MR_1] = 0$$

$$\text{(5-2)} \qquad E[MR_1] = b - \frac{\mathrm{cov}[U'(\pi),MR_1]}{E[U'(\pi)]}.$$

$$\frac{\partial E[U(\pi)]}{\partial Q_2} = E[U'(\pi)\{P_2(v) + Q_2 \frac{\partial P_2(v)}{\partial Q_2} - b - rk\}] = 0$$

$$= E[U'(\pi)\{MR_2 - b - rk\}] = 0$$

$$= E[U'(\pi)]E[MR_2] - E[U'(\pi)]b - E[U'(\pi)]rk$$

$$+ \mathrm{cov}\ [U'(\pi),MR_2] = 0$$

$$\text{(5-3)} \qquad E[MR_2] = b + rk - \frac{\mathrm{cov}[U'(\pi),MR_2]}{E[U'(\pi)]}$$

In order to make comparisons with the deterministic model set forth by Bailey and White [5] we need to express our results in terms of expected prices. Since

$$E[MR] = E[P + Q \frac{\partial P}{\partial Q}]$$

$$= E[P] + Q \frac{\partial P}{\partial Q}$$

$$= E[P][1 + \frac{Q}{E[P]} \frac{\partial P}{\partial Q}]$$

$$= E[P][1 - \frac{1}{E[\eta]}]$$

From equations (5-2) and (5-3) the pricing solutions for offpeak and peak demands when faced with uncertainty become, respectively

$$\text{(5-4)} \qquad E[P_1] = \frac{b}{1 - \frac{1}{E[\eta_1]}} - \frac{\mathrm{cov}[U'(\pi),MR_1]}{[1 - \frac{1}{E[\eta_1]}]E[U'(\pi)]}$$

$$= \left[b - \frac{\mathrm{cov}[U'(\pi),MR_1]}{E[U'(\pi)]} \right] \left[\frac{1}{1 - \frac{1}{E[\eta_1]}} \right]$$

and

$$(5\text{-}5)\qquad E[P_2] = \frac{b + rk}{1 - \frac{1}{E[\eta_2]}} - \frac{\operatorname{cov}[U'(\pi),MR_2]}{1 - \frac{1}{E[\eta_2]}\,E[U'(\pi)]}$$

$$= \left[b + rk - \frac{\operatorname{cov}[U'(\pi),MR_2]}{E[U'(\pi)]}\right]\left[\frac{1}{1 - \frac{1}{E[\eta_2]}}\right]$$

Since the covariance will, once again, be zero for the risk-neutral firm both its output and prices will be the same as in the deterministic model. Thus, on the average, $P^*_i = P_i^{B\&W}$ for the risk neutral firm, but if the density is summetric, $P^*_i > P_i^{B\&W}$ half of the time and $P^*_i < P_i^{B\&W}$ half the time.

If the firm is a risk-averter, we know from earlier analysis that the sign of the covariance will be negative and the capacity desired in this firm peak situation will be something less than the capacity required in the risk-neutral case as $E[MR_2] > b + rk$ (from equation 5-2). Assuming $E[\eta] = \eta_i$, prices charged by the risk-averse firm will be higher than the deterministic prices by the value of the term containing the covariance.

This is depicted graphically below as Figure 5-1:[3]

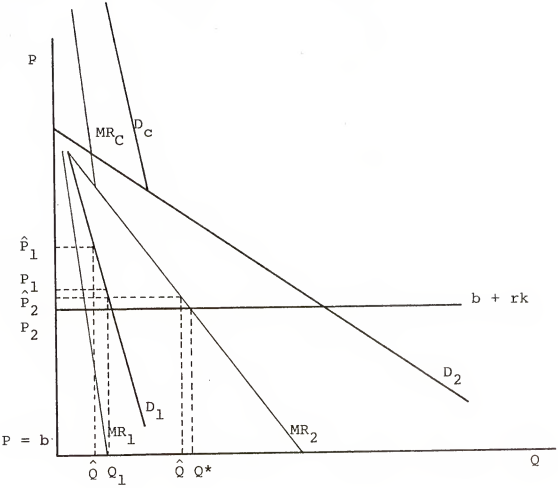


FIGURE 5-1

UNREGULATED FIRM PEAK

If $D_i = E(D_i)$, then $P^*_i > P_i^{B\&W}$. On the average, $P^*_i > P_i^{B\&W}$ for risk-averse firm.[4]

Offpeak prices are higher than peak prices for the unregulated firm maximizing profits and subject to uncertainty in the demand function whenever

$$\left[b - \frac{\operatorname{cov}[U'(\pi), MR_1]}{E[U'(\pi)]}\right]\left[\frac{1}{1 - \frac{1}{E[\eta_1]}}\right] > \tag{5-6}$$

$$\left[b + rk - \frac{\operatorname{cov}[U'(\pi), MR_1]}{E[U'(\pi)]}\right]\left[\frac{1}{1 - \frac{1}{E[\eta_2]}}\right]$$

This is the stochastic analog of the Bailey and White [4] result. If expected elasticities of peak and offpeak demands were equal, the offpeak prices would always be lower than peak prices because of their relationship to marginal costs. But it would be highly unlikely to have equal expected elasticities. Thus, as in the Bailey and White deterministic model, when offpeak demand is sufficiently more inelastic than peak demand so as to offset a larger portion of the margional costs attributable to the peak, offpeak price can be higher than peak price. Again, this pricing reversal is most likely to occur if marginal capacity costs are small relative to marginal operating costs.

Based on the uncertainty information acquired in earlier chapters and limiting to the firm peak case these pricing and output (capacity) solutions are not surprising. The more interesting case involves demand uncertainty and shifting peaks. The usual assumption of independent, equal length demands is made and the profit-maximizing firm's random profit function is

$$\pi = P_1(u)Q_1 + P_2(v)Q_2 - b(Q_1+Q_2) - rkQ^* \tag{5-7}$$

where $Q^* = Q_1 = Q_2$.

Because of demand uncertainty, we need to maximize the expected utility of profit in finding the pricing and output solutions. The firm's objective function is

$$E[U(\pi)] = E[U\{P_1(u)Q_1 + P_2(v)Q_2 - b(Q_1+Q_2) - rkQ^*\}]$$

The resultant first-order condition is

$$\frac{\partial E[U(\pi)]}{\partial Q^*} = E[U'(\pi)\{P_1(u) + Q_1 \frac{\partial P_1(u)}{\partial Q^*} + P_2(v)$$

$$+ Q_2 \frac{\partial P_2(v)}{\partial Q^*} - b \frac{\partial Q_1}{\partial Q^*} - b \frac{\partial Q_2}{\partial Q^*} - rk\}] = 0,$$

where $\frac{\partial Q_1}{\partial Q^*} = \frac{\partial Q_2}{\partial Q^*} = 1.$

Thus we may write

$$E[U'(\pi)\{MR_1 + MR_2 - 2b - rk\}] = 0$$

$$= E[U'(\pi)]E[MR_1] + E[U'(\pi)]E[MR_2] - 2bE[U'(\pi)]$$

$$- rkE[U'(\pi)] + \text{cov}[U'(\pi),MR_1] + \text{cov}[U'(\pi),MR_2] = 0$$

(5-8) $$E[MR_1] + E[MR_2] = 2b + rk$$

$$- \frac{\text{cov}[U'(\pi),MR_1] + \text{cov}[U'(\pi),MR_2]}{E[U'(\pi)]}$$

In the certainty case, the objective functions and its solution for the shifting peak situation are as follows

$$\pi = P_1Q_1 + P_2Q_2 - b(Q_1+Q_2) - rkQ^*$$

where $Q^* = Q_1 = Q_2$ and

$$\frac{\partial \pi}{\partial Q^*} = P_1 + Q_1 \frac{\partial P}{\partial Q^*} + P_2 + Q_2 \frac{\partial P_2}{\partial Q^*} - b - b - rk = 0$$

(5-9) $$MR_1 + MR_2 = 2b + rk, \text{ or}$$

$$P_1[1 - \frac{1}{\eta_1}] + P_2[1 - \frac{1}{\eta_2}] = 2b + rk$$

Rewriting equation (5-8) in terms of expected prices we find[5]

(5-8') $$E[P_1][1 - \frac{1}{E[\eta_1]} + E[P_2][1 - \frac{1}{E[\eta_2]}] = 2b + rk$$

$$- \frac{\text{cov}[U'(\pi),MR_1] + \text{cov}[U'(\pi),MR_2]}{E[U'(\pi)]}$$

The pricing solutions are depicted graphically below in Figure 5-2.

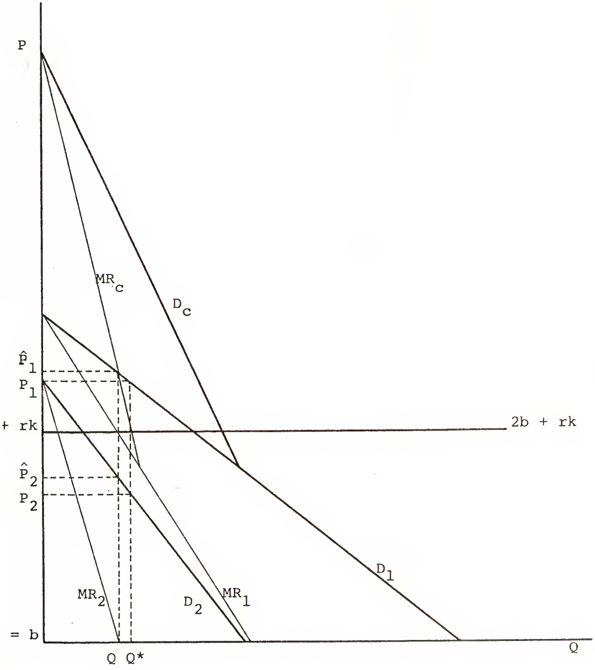


FIGURE 5-2

UNREGULATED SHIFTING PEAK

We can see from Figure 5-2 that output, Q*, is set where the aggregate expected marginal revenue curve intersects the combined operating and capacity cost curve. Thus, the output required by both demand customers will be Q^* but D_1 users will pay P_1 whereas D_2 users only need pay P_2. When the firm is risk-averse it will produce something less than Q^*, say Q. Here prices for D_1 and D_2 users will be P_1 and P_2, respectively.[6] The divergence between the prices depends on the relative intensities and elasticities of the demands as well as the degree of risk-aversion.

In Chapter 4 we saw that the profit maximizing firm subject to rate of return regulation in the deterministic model had the following pricing solutions

$$(4\text{-}22) \qquad \overline{P}_G = \frac{b + rk - \frac{\lambda}{1-\lambda}(s-r)K}{1 - 1/\overline{\eta}}$$

$$\underline{P}_G = \frac{b}{1 - 1/\underline{\eta}}$$

Extending this first to the firm peak case facing uncertainty we can express the objective function as it is in equation (5-6) and subject it to the following expected rate of return constraint

$$(5\text{-}10) \qquad E[P_1(u)Q_1 + P_2(v)Q_2 - b(Q_1+Q_2) - skQ^*] \leq 0$$

Using equations (5-6) and (5-10) we can formulate the following expression

$$(5\text{-}11) \qquad E[U(\cdot)] = E[U\{P_1(u)Q_1 + P_2(v)Q_2 - b(Q_1 + Q_2) - rkQ^* - \lambda(P_1Q_1 + P_2Q_2 - b(Q_1+Q_2) - skQ^*)\}],$$

where

λ = the parameter associated with the rate of return constraint,

$Q^* = Q_2$, and

$Q(\cdot)$ = utility of constrained profits.

Differentiating we have

$$\frac{\partial E[U(\cdot)]}{\partial Q_2} = E[U'(\cdot)\{P_2(v) + Q_2 \frac{\partial P_2(v)}{\partial Q_2} - b - rk$$

$$- \lambda(\bar{P}_2 + Q_2 \frac{\partial P_2}{\partial Q_2} - b - sk)\}] = 0$$

$$= E[U'(\cdot)\{MR_2 - b - rk - \lambda MR_2 + \lambda b + \lambda sk\}] = 0$$

$$E[U'(\cdot)\{(1-\lambda)(MR_2 - b - rk + \frac{\lambda}{1-\lambda}(s-r)k)\}] = 0$$

$$(1-\lambda)\{E[U'(\cdot)]E[MR_2] - bE[U'(\cdot)] - rkE[U'(\cdot)]$$

$$+ \frac{\lambda}{1-\lambda}(s-r)kE[U'(\cdot)] + \mathrm{cov}[U'(\cdot),MR_2]\} = 0$$

$$E[MR_2] = b + rk - \frac{\lambda}{1-\lambda}(s-r)k - \frac{\mathrm{cov}[U'(\cdot),MR_2]}{E[U'(\cdot)]} \tag{5-12}$$

and

$$\frac{\partial E[U'(\cdot)]}{\partial Q_1} = E[U'(\cdot)\{P_1(u) + Q_1 \frac{\partial P_1(u)}{\partial Q_1} - b$$

$$- \lambda(P_1 + Q_1 \frac{\partial \bar{P}_1}{\partial Q_1} - b)\}] = 0$$

$$E[U'(\cdot)\{MR_1 - b - \lambda MR_1 + \lambda b\}] = 0$$

$$E[U'(\cdot)\{(1-\lambda)(MR_1 - b)\}] = 0$$

$$(1-\lambda)[EU'(\cdot)]E[MR_1] - bE[U(\cdot)]$$

$$+ \mathrm{cov}[U'(\cdot),MR_1] = 0$$

$$E[MR_1] = b - \frac{\mathrm{cov}[U'(\cdot),MR_1]}{E[U'(\cdot)]} \tag{5-13}$$

Graphically these pricing and output decisions can be viewed in Figure 5-3 below:

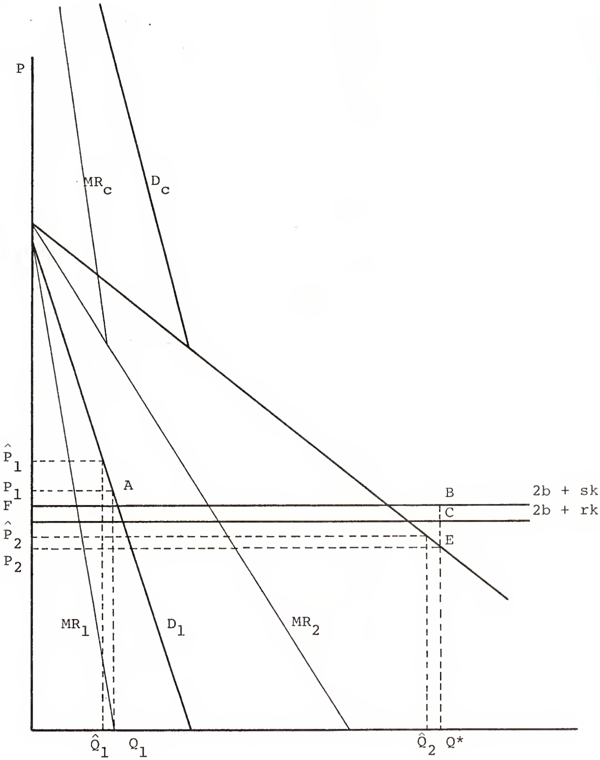


FIGURE 5-3

REGULATED FIRM PEAK

For the risk-neutral firm the offpeak price is identical to the deterministic model's offpeak price and its output is set where MR = b since demand does not press on capacity. The peak price and output (capacity) is derived then in the same manner as it was earlier. That is, the surplus profits, P_1AQ_1b, accrued in the offpeak demand can subsidize a deficit in the peak demand equal to that amount. The peak price in this case would be P_2 while the output (capacity) would be Q^*. Because the offpeak surplus profits are greater than the profits allowed, the firm is induced to price their peak demand below its cost and expand its capacity to take advantage of the offpeak surplus.

If the firm were risk-averse, the output for the offpeak demand would be something less than Q_1, say $\hat{Q}_1$, since $E[MR_1] > b$. Then the offpeak price would rise and the offpeak surplus profit would fall. (This is assuming the marginal revenue and marginal cost curves for offpeak demand intersect on the elastic portion of its demand curve. This is a reasonable assumption for our purposes.) With a reduced offpeak surplus, the peak deficit must correspondingly be reduced. This will mean an increase in the peak price, P_2 to $\hat{P}_2$ and a decrease in the output (capacity) from $Q^* = Q_2$ to $\hat{Q}_2$.[7]

Applying the same analysis to the shifting peak case we find by maximizing the expected utility of constrained profits for the firm using equations (5-6) and (5-10) but where $Q^* = Q_1 = Q_2$.

$$(5\text{-}14) \quad E[U(\cdot)] = E[U\{P_1(u)Q_1 + P_2(v)Q_2 - b(Q_1+Q_2) - rkQ^*$$

$$- \lambda(\overline{P}_1Q_1 + \overline{P}_2Q_2 - b(Q_1+ Q_2) - skQ^*)\}]$$

$$\frac{\partial E[U(\cdot)]}{\partial Q^*} = E[U'(\cdot)\{P_1(u) + Q_1 \frac{\partial P_1(u)}{\partial Q^*} + P_2(v)$$

$$+ Q_2 \frac{\partial P_2(v)}{\partial Q^*} - 2b - rk$$

$$- \lambda(\overline{P}_1 + Q_1 \frac{\partial \overline{P}_1}{\partial Q^*} + P_2 + Q_2 \frac{\partial \overline{P}_2}{\partial Q^*} - 2b - sk)\}]$$

$$= E[U'(\cdot)\{MR_1 + MR_2 - 2b - rk - \lambda MR_1 - \lambda MR_2$$

$$+ 2b\lambda + \lambda sk\}] = 0$$

$$= E[U'(\cdot)\{(1-\lambda)MR_1 + (1-\lambda)MR_2 - (1-\lambda)2b$$

$$- (1-\lambda)rk + \lambda(s-r)k\}] = 0$$

$$= E[U'(\cdot)\{[1-\lambda][MR_1 + MR_2 - 2b - rk$$

$$+ \frac{\lambda}{1-\lambda}(s-r)k]\}] = 0$$

$$= (1-\lambda)E[U'(\cdot)\{MR_1 + MR_2 - 2b - rk$$

$$+ \frac{\lambda}{1-\lambda}(s-r)k\}] = 0$$

$$= (1-\lambda)E[U'(\cdot)]E[MR_1] + E[U'(\cdot)E[MR_2]$$

$$+ \mathrm{cov}[U'(\cdot),MR_1] + \mathrm{cov}[U'(\cdot),MR_2]$$

$$- E[U'(\cdot)]2b - E[U'(\cdot)]rkE[U'(\cdot)]\frac{\lambda}{1-\lambda}(s-r)k] = 0$$

$$(5\text{-}15) \quad E[MR_1] + E[MR_2] = 2b + rk - \frac{\lambda}{1-\lambda}(s-r)k$$

$$- \frac{\mathrm{cov}[U'(.),MR_1] + \mathrm{cov}[U'(.),MR_2]}{E[U'(\cdot)]}$$

Rewriting equation (5-15) in terms of expected prices gives us

$$(5\text{-}15') \quad E[P_1][1 - \frac{1}{E[\eta_1]}] + E[P_2][1 - \frac{1}{E[\eta_2]}] = 2b + rk - \frac{\lambda}{1-\lambda}(s-r)k - \frac{\mathrm{cov}[U'(\bar{\cdot}), MR_1] + \mathrm{cov}[U'(\bar{\cdot}), MR_2]}{E[U'(\bar{\cdot})]}$$

As expected, assuming $E[\eta_i] = \eta_i$ and $D_i = E[D_i]$, the regulated risk-neutral firm will produce the same capacity output and charge the same price on the average as the firm in the deterministic model.[8]

Again, the risk averse firm will have negative covariance making $E[MR_1] + E[MR_2] > 2b + rk - \frac{\lambda}{1-\lambda}(s-r)k$ while the capacity desired will be something less than the capacity required in the risk-neutral case. The pricing solutions for these firms are depicted in Figure 5-4.

In the shifting peak case, offpeak surpluses will not exist and the regulated deterministic firm (or the regulated risk-neutral firm) will set capacity where the aggregate (expected) price level will generate the allowed rate of return. In Figure 5-4 this will be where $P = 2b + sk$ where D_1 users will pay a price, P_1, for their output Q^* and D_2 users pay only P_2 for the same output Q^*. Intuitively we can see the area P_CAQ^*b allows the firm to recover the market cost of its capital, bP_1 per unit as well as an additional P_CP_1 per unit of capital (due to $s>r$).

The pricing differential between the two peak demands corresponds to their intensity and elasticity of demand.

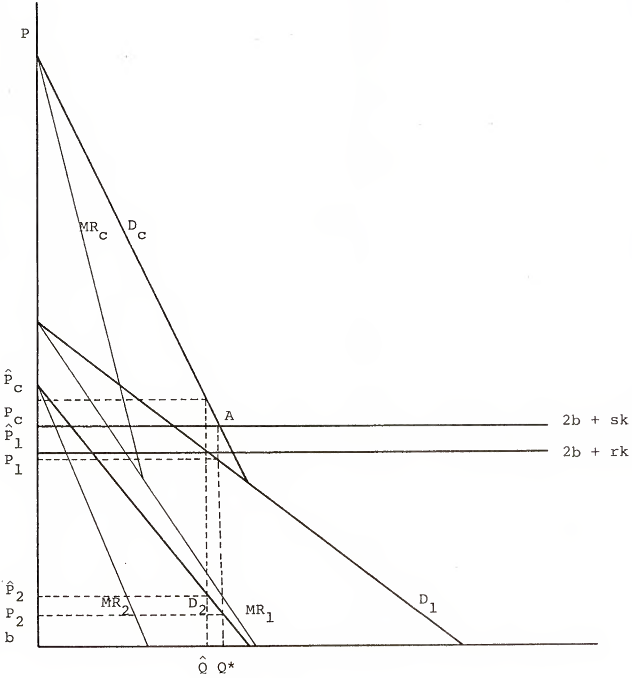


FIGURE 5-4

REGULATED SHIFTING PEAK

If the marginal capacity costs were higher, and the difference between s and r remained constant then both 2b + sk and 2b + rk would shift upward and the difference between the peak prices would be reduced.

If the firm were risk-averse, we now know that it will choose to produce at a capacity somewhere below the risk-neutral firm's level Q*, say $\hat{Q}$. Figure 5-4 shows the prices to rise to $\hat{P}_1$ and $\hat{P}_2$ where $\hat{P}_2$ experiences a more substantial increase because of the value of η_2 in equation (5-15').

With the risk-taking firm we know that the results should lead one to expect, on the average, lower prices and increased capacity.

NOTES

[1] The welfare maximizing sollution has already been solved by Brown and Johnson

[2] The Steiner [24] assumption of equal length demands is utilized in this analysis, although it is recognized that Williamson's [28] weighting demands and finding an effective demand for capacity curve is the more generalized form. The equal length demand assumption is strictly to facilitate the graphical presentations.

[3] In practice, the marginal operating cost is not zero. The values of $E[MR_i]$, b, rk have been arbitrarily selected for graphical convenience. Also the subscripts 1 and 2 represent offpeak and peak demand, respectively.

[4] The reverse holds for the risk-taking firm. That is, capacity required will be larger and prices lower than in the certainty case.

[5] The point at which the long-run marginal cost curve intersects the aggregate marginal revenue curve will be an important factor in determining the degree of pricing divergence for the nonlinear risk preferent firms.

[6] The reverse analysis holds for the risk-taking firm.

[7] Again, the reverse analysis holds for the risk-taking firm.

[8] By this time it is quite clear that by dropping the expectations and covariance terms in equations (5-15) and (5-15') we have the equations for the certainty case.

CHAPTER 6

SUMMARY AND CONCLUSIONS

This dissertation set out to formulate some uncertainty-related models for use in analyzing both the behavioral problems and the peak load pricing problems of the firm. We found that the manner in which uncertainty and the ex ante decision variables are introduced into the model will alter the outcome of the firm's decisions. Chapters 2 and 4 provided detailed discussion of the Averch-Johnson and peak-load pricing literature. Chapter 3 analyzed the prices, outputs, and input proportions under the various assumed conditions of uncertainty and firm attitudes toward risk. This analysis was considered in both the unregulated and regulated context. Chapter 5 found the traditional peak load pricing principles inadequate when a rate of return on capital constraint and an uncertainty variable were taken into consideration.

Yet it is difficult to escape the fact that this mathematical formulation of the model is an inadequate description of the regulatory process. Several important aspects of regulation have not been incorporated into the model. For example, since public utilities are required to meet all demand requirements any quantity-setting decisions by the firm are limited. Additionally, the models here ignore the fact that regulatory commissions have a great deal of control over prices, thus limiting the scope of the firm's price-setting flexibility.

Several important regulatory issues, such as quality of service or regulatory lag, cannot be dealt with because of the static nature of these models. The regulatory lag question has been examined by Bailey and Coleman [3] by introducing a discrete interval before regulatory action is taken. Thus, although it was beyond the scope of this paper the possibility exists for lag to be incorporated into uncertainty-related models and its optimal responses examined. A more realistic approach to formulating the model may also mean the inclusion of environmental as well as technological growth considerations. That is, the present models ignore the possibility that the regulated firm may undertake research and development and in so doing affect the capital-labor input ratio.

Also, in practice, public utilities do not employ single rate price structures but rather use some form of multipart pricing tariffs. Further research might be made in the integration of uncertainty with customer and usage charges. Furthermore, empirical evidence may exist as to the importance of potential shifting peaks and these findings may affect future pricing policy prescriptions.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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